Optimization

ID: 9610

Time required
45 minutes

Activity Overview

Students will learn how to use the second derivative test to find maxima and minima in word problems and solve optimization in parametric functions.

Topic: Applications of Derivatives

- *Find the maximum or minimum value of a function in an optimization problem by finding its critical points and applying the second derivative test. Use **Solve** (in the Algebra menu) to check the solution to f'(x)=0.*
- *Use the command **fMin** or **fMax** to verify a manually computed extremum.*
- *Solve optimization problems involving parametric functions.*

Teacher Preparation and Notes

- *This investigation uses **fMax or fMin** to answer questions. Students will also have to be able to take derivatives and solve equations on their own.*
- *This activity is designed to be student-centered with the teacher acting as a facilitator while students work cooperatively.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files), go to education.ti.com/exchange and enter "9610" in the quick search box.***

Associated Materials

- *Optimization_Student.doc*
- *Optimization_Soln.tns*
- *Optimization.tns*

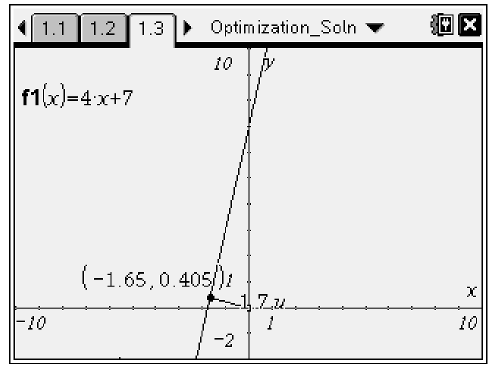
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Application of the Derivative (TI-89 Titanium) — 4275*
- *Find Optimization Points with Derivatives (TI-89 Titanium) — 3239*
- *Where Should it Go? Optimization Exercise (TI-Nspire CAS technology) — 10204*
- *Maximizing Area (TI-Nspire technology) —10043*
- *Design a Better Drink Can (TI-Nspire technology) — 8272*

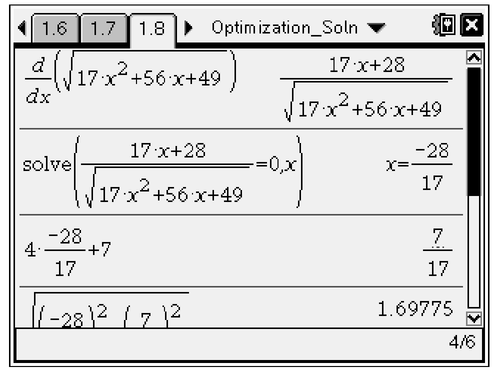
Problem 1 – Optimization of distance and area

On page 1.3, students will graph the equation $y = 4x + 7$. To begin their investigation, students construct a point on the line and a segment connecting the point to the origin.

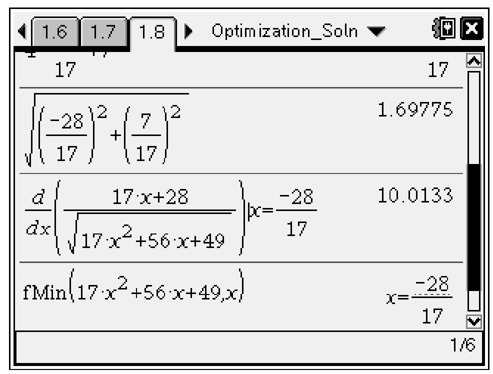


After students find the length of the segment, they should move the point along the line to estimate the minimum distance between the two points.

Students need to minimize the function $s = \sqrt{x^2 + y^2}$ where x and y are the coordinates of a point on the line. The constraint is the equation of the line. They should rewrite the function using one variable: $s = \sqrt{x^2 + (4x+7)^2} = \sqrt{17x^2 + 56x + 49}$.

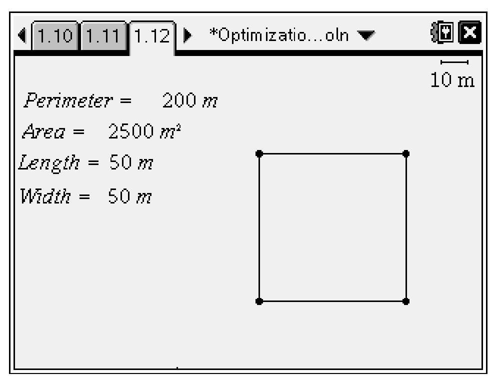


On page 1.8, to find the exact coordinates of the point, students will take the first derivative (**MENU > Calculus > Derivative**), and solve for the critical value (**MENU > Algebra > Solve**), and take the second derivative. Since the second derivative is always positive, there a minimum at the critical value of $x = -\frac{28}{17}$.



To find the y-coordinate, students should substitute the value of x into the original equation $y = 4x + 7$. To find the distance, they should substitute the x- and y-values into the function $s = \sqrt{x^2 + y^2}$. The point is (–1.647, 0.412) and the distance is 1.698 units.

Students should find the second derivative to confirm the point is a minimum. They can also confirm the x-value of the minimum point using the **fMin** command, as shown to the right.



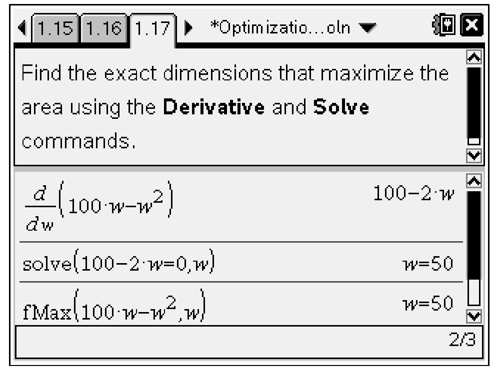
On page 1.9, students will construct a rectangle with a perimeter of 200 m. To lock the perimeter, use the **Attributes** tool, click on the perimeter measurement, move down to the lock, press right to lock, and press enter. Once they have locked the measurement, students can find the measurements of the length, width, and area of the rectangle. Students should estimate the dimensions to maximize the area.

Students are to maximize the function $A = l \cdot w$. The constraint is $2l + 2w = 200$. Since $l = 100 - w$, students can rewrite the function as $A = (100 - w) \cdot w = 100w - w^2$.

Students will take the first derivative and solve to find the critical value is $w = 50$. The second derivative is always negative, so we have a maximum.

When $w = 50$ m, $l = 50$ m. The maximum area is 2500 m^2.

Students can use the **fMax** command from the Calculus menu to confirm the value of w.



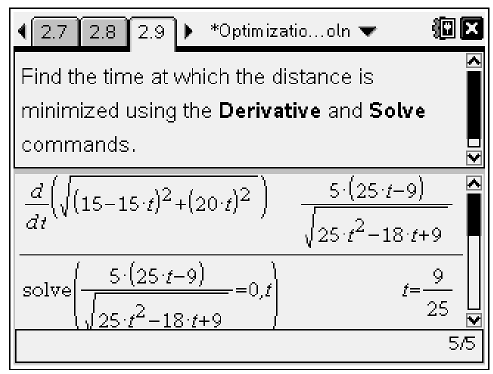
> **TI-Nspire Navigator Opportunity:** ***Screen Capture*** **and/or** ***Live Presenter***
>
> **See Note 1 at the end of this lesson.**

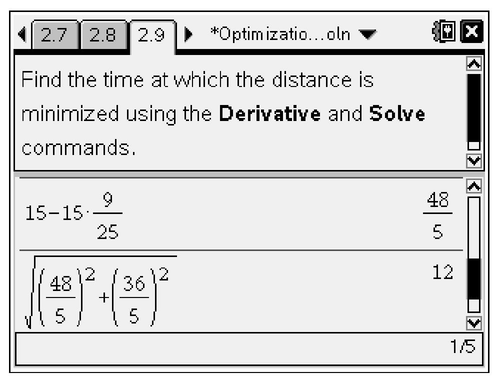
Problem 2 – Optimization of time derivative problems

On page 2.1 students will see a diagram of the problem. Remind students to use t, for time, instead of x. The position equations are the constraints.

The boat heading north is going from the right angle to the point northward. Its position equation is $y = 20t$.

The boat heading west is going to the right angle. At 1 pm, it is one hour from the arrival time 2 pm, so it is 15 km away. Its position equation is $x = 15 – 15t$.





Students should minimize the distance function $d = \sqrt{x^2 + y^2} = \sqrt{(15 - 15t)^2 + (20t)^2}$.

There is a restriction of $0 < t < 1$ because the boats are only moving for 1 hour. Students will solve the first derivate to find the critical time is ***t* = 9/25**. Because the second derivative is always positive, there is a minimum.

The time at which the distance between the boats is minimized is (9/25) · 60 = 21.6 minutes after 1 pm, or about 1:22 pm. The distance between the two boats is 12 km.

> **TI-Nspire Navigator Opportunity:** ***Screen Capture***
>
> **See Note 2 at the end of this lesson.**

Extension – Parametric function

On page 3.2, students are to graph the parametric equations found on the previous page. To rewrite the parametric equations, students will need to know that sin(30°) = 0.5 and cos(30°) = $\frac{\sqrt{3}}{2}$.

To find when the projectile hits the ground, have students set $y = 0$ and solve ($t = 0$ and $t = 51.02$). Substituting these values into the x function gives how far away it lands (22.092.5 units). Students can find the maximum height when $dy/dt = 0$ ($t \approx 25.51$). Substituting this value into the function for y, they should obtain a height of 3188.78 units.

TI-Nspire Navigator Opportunity: *Screen Capture*

See Note 3 at the end of this lesson.

TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Screen Capture and/or Live Presenter*
You can use Screen Capture during this problem to verify students are following along with the lesson. You may also choose one or more students to be presenters and lead other students through the activity.

Note 2
Problem 2, *Screen Capture*
You may choose to use screen capture to verify students are able to use the **Derivative** and **Solve** commands to complete the activity.

Note 3
Entire Document, *Quick Poll*
You may choose to use Quick Poll to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask.