

Complex Numbers

ID: 10887

 Time required
 15 minutes

Activity Overview

In this activity, students calculate problems from the student worksheet to determine the rules for adding, subtracting, multiplying and dividing complex numbers.

Topic: Complex Numbers

- *Operations with Complex Numbers*

Teacher Preparation and Notes

- *Students have space on the handout to write solutions. If you are using Connect-to-Class, students can insert a Notes Application Page (Home: Notes) and type their explanations for collection.*
- *Multiplying by the complex conjugate is not a part of this activity. It is a natural extension for after this activity.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "10887" in the keyword search box.***

Associated Materials

- *ComplexNumbers_Student.doc*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Complex Numbers: Plotting and Polar Form (TI-Nspire technology) — 8908*
- *Steady State Circuit Analysis & Filter Design (TI-89 Titanium) — 3063*

Students will enter problems in their calculators and record answers on the student worksheet. Students should work in pairs or small groups in order to help look for patterns and determine an algorithm for operations with complex numbers.

Remind students that i is found by pressing $\boxed{2\text{nd}} + [i]$.

Adding Complex Numbers

Students will use their calculator to find an algorithm for adding complex numbers and solve some addition problems. Encourage students to try solving the problems on their own before entering the problems in the *Calculator* application. Students may mention that adding two complex numbers is similar to adding two binomial expressions, such as $(2 + 3x) + (5 - 4x)$

$3+4i+2+5i$	$5+9i$
$1-6i+3-2i$	$4-8i$
$2+5i+6-8i$	$8-3i$

Subtracting Complex Numbers

Students should discover that subtracting complex numbers is similar to adding complex numbers.

$(3+4i)-(2+5i)$	$1-i$
$(1-6i)-(3-2i)$	$-2-4i$
$(2+5i)-(6-8i)$	$-4+13i$

Multiplying Complex Numbers

Students will move on to multiplying complex numbers. If students have trouble seeing why there is no i^2 term in the answers, encourage them to work through the problems by hand and use $\sqrt{-1}$ instead of i . Students should see $i^2 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = -1$.

$(2+5i)(6-8i)$	$52+14i$
$(-2+3i)(1-2i)$	$4+7i$
$(4-3i)(-5-7i)$	$-41-13i$

Dividing Complex Numbers

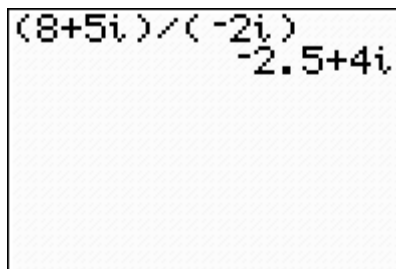
Students will divide a complex number by another complex number. In order to explain why i is not in the denominator of the answers, tell students to think of possible ways to eliminate the i in the denominator. Students should recall that $i^2 = -1$. What happens if this was changed to $i \cdot -i$? ($i \cdot -i = -(i \cdot i) = -(-1) = 1$)

$(2+4i) \div (3i)$	$1.333333333-.666666667i$
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Because the numerator and denominator have both been multiplied by i . This results in an i^2 in the denominator that is then replaced with -1 .

$$\frac{8+5i}{-2i} \cdot \frac{i}{i} = \frac{8i+5i^2}{-2i^2} = \frac{8i+5(-1)}{-2(-1)} = \frac{8i-5}{2} = \frac{8}{2}i - \frac{5}{2} =$$

$$4i - 2.5 = -2.5 + 4i$$



$(8+5i) / (-2i)$
 $-2.5+4i$

Solutions – Student worksheet

Adding Complex Numbers

1. $5 + 9i$
2. $4 - 8i$
3. $8 - 3i$
4. $-1 + i$
5. $-1 - 10i$
6. Sample answer: when adding two complex numbers, add the real parts together and then add complex parts together.

Subtracting Complex Numbers

1. $1 - i$
2. $-2 - 4i$
3. $-4 + 13i$
4. $-3 + 5i$
5. $9 + 4i$
6. Sample answer: when subtracting two complex numbers, subtract the real parts and then subtract complex parts.

Multiplying Complex Numbers

1. $-14 - 23i$
2. $-9 - 20i$
3. Sample answer: there is no i^2 in the answers because $i^2 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = -1$
4. $52 + 14i$
5. $4 + 7i$
6. $-41 - 13i$
7. Sample answer: when multiplying two complex numbers, apply the Distributive Property, and replace i^2 with -1 , and simplify.

Dividing Complex Numbers

1. $\frac{4}{3} - \frac{2}{3}i$

2. $-1 - \frac{1}{2}i$

3. Sample answer: I can multiply by $\frac{i}{i}$ to eliminate the imaginary number in the denominator.

4. $\frac{-3}{4} - \frac{1}{2}i$

5. $\frac{7}{3} + \frac{4}{3}i$

6. $\frac{-5}{2} + 4i$

7. Again, students should multiply the expressions by $\frac{i}{i}$ to establish a pattern.

8. Sample answer: Students should begin seeing a pattern. For example,
 $\frac{(2-3i)}{4i} = \frac{(2-3i)}{4i} \times \frac{i}{i} = \frac{(-2i-3i^2)}{4i^2} = \frac{(-2i-3(-1))}{4(-1)} = \frac{(-2i+3)}{-4} = \frac{(-3-2i)}{4}$. The numbers in the numerator changes positions with i remaining in the second term of the numerator and change the sign of the new second term. Then, i is no longer in the denominator because multiplying i by i gives -1 .

9. **CHALLENGE:**

$$\frac{3+4i}{2-5i} \cdot \frac{2+5i}{2+5i} = \frac{6+15i+12i+20i^2}{4+10i-10i-25i^2} = \frac{6+27i-20}{4+25} = \frac{-14+27i}{29} = \frac{-14}{29} + \frac{27}{29}i$$