

# REPEATED DILUTION OF DIFFUSELY HELD DEBT

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## Abstract

Debt with many creditors is analyzed in a continuous-time pricing model of the levered firm in the presence of corporate taxes. We specifically allow for debtor opportunism in form of repeated strategic renegotiation offers and default threats. Dispersed creditors will only accept coupon concessions in exchange for guaranteed liquidation rights, e.g. collateral. The ex ante optimal debt contract is secured with assets which gradually become worthless as the firm approaches the preferred liquidation conditions, in order to allow for sufficient, but delayed renegotiability. Compared with single-creditor debt, dispersed debt offers a larger debt capacity, and it is preferable ex-ante if the value of collateralizable assets is then reduced. Our model can explain credit risk premia in excess of those supported by a single creditor model with opportunistic renegotiation.

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Recently, a growing body of literature has introduced corporate finance concepts into valuation models of defaultable securities. Variables such as the capital structure choice, the lower reorganization bound and the outcome of bargaining between debtor and creditors have been endogenized in fully dynamic models.<sup>1</sup> Yet in all the existing work incorporating capital structure theory or bargaining models into debt valuation theory, the number of creditors has been ignored and implicitly, a fiction has been invoked that the borrower is confronted with a single “representative” creditor.

The purpose of this paper is to explicitly model the strategic interaction between shareholders and creditors when there are *multiple creditors*. We study dynamic strategies of debt renegotiation and default in this environment and analyze the impact of the optimal opportunistic debtor strategy on the value of defaultable bonds and on financing decisions.

There is little reason to assume that creditors would coordinate their responses to a renegotiation offer or a default threat: An individual creditor will prefer to free-ride on the debt restructuring effort of others, and the larger the number of creditors, the stronger this tendency to hold out.<sup>2</sup> Individual creditors are not inclined to make concessions, although they realize that doing so would be in their collective interest. The importance of the hold-out effect is highlighted by numerous empirical studies showing that out-of-court debt restructurings with many creditors bear a substantial risk of failure.<sup>3</sup>

This issue plays a prominent role in recent literature on the choice between private (or concentrated) and public (or dispersed) debt, which has discussed the advantages of sticky renegotiation. Bolton and Scharfstein (1996) and Berglöf and von Thadden (1994) argue that the lack of renegotiability can be an important advantage of dispersed debt, since it makes strategic default less attractive and mitigates risk-shifting incentives. Similarly, Dewatripont and Maskin (1995) and Bolton and Freixas (2000) argue that some firms will deliberately disperse their debt in order to signal their commitment to good behavior. Focusing on differences in how dispersed or concentrated debt claims are monitored, Diamond (1991), Rajan (1992) and Chemmanur and Fulghieri (1994) argue that some firms will prefer to borrow at arm’s length in order to signal their strength or to avoid the pitfalls of an exclusive lending relationship.

Diffusely held debt is not simply immune to renegotiation efforts. But if creditors are dispersed, debt restructuring proposals must be engineered so as to spoil the attractiveness of

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<sup>1</sup>Building on Black and Cox (1976), Leland (1994) and Leland and Toft (1996) endogenize the shareholders’ decision to trigger liquidation and determine the optimal capital structure of firm. Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997) and Mella-Barral (1999) extend the analysis to allow for the strategic interaction between shareholders and debtholders in debt renegotiation, prior to liquidation. Ericsson (2000) and Leland (1998) provide models that account for the asset substitution or risk shifting effect. Goldstein, Ju and Leland (2001) further consider releverage opportunities.

<sup>2</sup>More precisely, an individual investors’ incentives to hold out depends on the probability of being “pivotal” for success or failure of the tender offer, quite similar to the analogous effect in takeover bids. See e.g. Detragiache and Garella (1996) and Hege (2002) for an analysis.

the hold-out option. Opportunistic shareholders faced with a non-cohesive group of creditors have actually powerful devices at hand to dilute the value of creditors rejecting the offer.

These dilution devices have in common that they *(i)* impose a scheme of wealth transfers *from creditor to creditor*, and *(ii)* make these transfers implicitly *conditional* on rejection of the debt restructuring proposal. These strategies are thus not applicable if the debt issuer faces a single creditor, and they are coercive since creditors stand to lose if they do not accept the restructuring proposal, relative to those who do. Creditors are made to rush in to tender, in particular if the number of new contracts is limited and they are served on a first-come-first-serve basis.

Empirical literature suggests that debt-for-debt exchange offers proposing more liquidation rights are the leading case of such dilution threats and are in fact very common. A well-known example are the so-called “exit consents”, where the right to participate in the exchange or tender offer is explicitly tied to a vote approving the exit from a covenant restricting the issuance of new debt (Roe (1987)). Bondholders will then first rush in to waive the covenant to secure their right to exchange; once the covenant is stripped, each bondholder prefers to tender because if he were the only one to hold out, the liquidation value of his claim, as well as the secondary market value of a severely illiquid bond issue, would suffer.

This paper examines the optimal debt renegotiation strategy of an opportunistic debtor facing a non-coordinated group of creditors in a continuous-time model of the levered firm. The set-up of the model is adapted from Mella-Barral (1999) to allow for multiple creditors and the tax advantage of debt. We allow for a rich set of actions at the discretion of the debtor and study strategies of repeated debt exchange offers and dilution threats. The exchange offer strategies available to shareholders follow closely typical procedures in debt exchange offers, allowing the debtor to offer more liquidation rights in exchange for concessions, as well as the possibility to default strategically. Importantly, these opportunistic actions can be taken at any time, and as often as the debtor likes.

The dynamic dimension turns out to be crucial: The possibility of subsequent renegotiation rounds severely limits the size of concessions that can be obtained. That is, an exchange offer cannot succeed if tendering creditors are only offered liquidation rights which can be re-expropriated later on. To be successful, it must offer additional liquidation rights which are *guaranteed*, in the sense that they are immune to subsequent dilution.

We solve for the shareholder’s ex post optimal exchange offer strategy, and show that the shareholder will successively trade coupon concessions for increases in guaranteed liquidation rights, until all expected liquidation proceeds are fully impaired by guaranteed liquidation

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<sup>3</sup>Empirical work by Brown, James and Mooradian (1993)(1994), James (1995)(1996) Franks and Torous (1989)(1994), Gilson, John and Lang (1990), Gilson (1997), Asquith, Gertner and Scharfstein (1994), Helwege (1999), Chatterjee, Dhillon and Ramirez (1995) and Hotchkiss (1995) shows evidence in this respect.

rights. Creditors' initial entitlement to a share of the liquidation proceeds which are not guaranteed turns out to have simply no value since it will subsequently be expropriated.

Moving backwards in time, we examine the optimal ex ante policy of the firm. The optimal debt contract will withhold the entire value of the liquidation rights at the anticipated abandonment point for use in contingent renegotiation. The optimal capital structure trades off the fiscal advantage of debt against an increasingly inefficient abandonment decision. We derive closed-form solutions for the value of equity and defaultable bonds.

We then compare the characteristics of renegotiable dispersed debt to those of debt issues that cannot be renegotiated, as they have been studied in Merton (1974) type models, notably by Leland (1994). We observe that if all assets are initially guaranteed in our setting, then dispersed debt is not renegotiable, facilitating this comparison. We find that the option to renegotiate debt is always valuable, since it allows to issue more debt ex ante.

We explore also the choice between dispersed (public) debt and debt held by a single creditor (private debt). Debt renegotiation models with a single creditor<sup>4</sup> show that shareholders, when faced with a single creditor, can strategically obtain concessions by threatening to walk away. We find that creditor dispersion enhances the ex ante borrowing capacity of the firm, because it credibly limits the size of concessions the opportunistic debtor can obtain ex post. As a result, the ex-ante optimal leverage of the firm and its debt tax shield are typically larger with dispersed debt, but single creditor debt guarantees an efficient ex post decision. The smaller the initial debt capacity of concentrated debt, the more attractive is the issue of dispersed debt.

Finally, we undertake an implementation of our model for a standard parameter structure with simple closed form solutions, and we perform a numerical simulation by means of an example. This analysis confirms that the optimal leverage ratio and the resulting credit spreads are very sensitive with respect to the model specification. It is numerically important to correctly account for the presence of *multiple creditors* and/or *initial collateral* or other guaranteed liquidation rights when choosing the bond valuation model.

As a result of the high optimal leverage ratio typically obtained with dispersed debt, our model is capable of explaining substantial credit risk premia under realistic parameter assumptions, and in excess of the risk premia obtained with a single creditor. Creditor dispersion may thus be one of the factors helping to explain why Merton-type debt models typically fail to generate credit spreads of the magnitude that is observed in practice. Interestingly, while the issue of dispersed debt dramatically affects the optimal financial structure and implied credit risk, the benefits in terms of overall firm value appear to be much more modest.

We present the set-up in Section I. In Section II, we define exchange offer strategies and

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<sup>4</sup>See Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997) and Mella-Barral (1999).

explain the mechanism of dilution. In Section III, we solve for the shareholder’s ex post optimal strategy. In Section IV, we examine the consequences for the creditors’ willingness to lend at entry. We determine the ex ante optimal debt contracts and capital structure of the firm. In Section V, we compare with the cases of non-renegotiable and of single-creditor debt. In Section VI, we provide closed form solutions and study a numerical example to assess the quantitative importance of the distinction we analyzed. Section VII documents that our model is consistent with empirical evidence on the use of dilution threats in practice. Section VIII looks at possible extensions and Section IX concludes.

## I. The Model

### A. Operations and the Abandonment Decision

Consider a firm and its real assets, controlled by a person, called the manager, from date  $t = 0$  on. The cash generating ability of the firm’s assets is related to a single uncertain state variable,  $x_t$ , which summarizes economic fundamentals, and follows a diffusion process:

$$dx_t = \mu(x_t)dt + \sigma(x_t)dB_t, \quad (1)$$

where  $B$  is a standard Brownian motion. Once the firm is set up, the manager can do the following:

1. She can generate a period income flow, combining her human capital and protected technology with the purchased real assets. Let  $\Pi(x_t)$  denote the pre-tax present value of a perpetual claim on the income flow that results from such operations, assuming no limited liability.
2. Although she could operate the firm forever, she can also *abandon* operations. We denote by  $V^*(x_t)$  the liquidation value of the firm’s real assets, net of bankruptcy costs.

We assume that an abandonment decision is *irreversible*.<sup>5</sup> Furthermore, there are some states of the world  $x$  where other parties, like competitors, have a better use for the assets than the incumbent. In these poor states,  $V^*(x)$  is actually greater than  $\Pi(x)$  and the abandonment decision is desirable, as formalized by Assumption 1 below. We assume that  $\Pi(x)$  is increasing in  $x$ ; this is not necessarily the case for  $V^*(x)$ .

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<sup>5</sup>Abandoning is akin to invoking the formal liquidation bankruptcy procedure (Chapter 7 of the US Bankruptcy Code of 1978). However, the set-up allows for a wider interpretation of the abandonment decision, without necessarily referring to the formal bankruptcy code. It allows, for example, to capture aspects of the property rights view of the firm: abandonment means then that relation-specific investments with a reduced value outside the firm are dismantled and parts of the cash generating ability of the firm is lost, and irreversibility means that the restoring of the combination of human and physical capital, after a period of abandonment, is not costless.

## B. Unlevered Value of the Firm

It is convenient to begin the analysis deriving the value of the firm if no debt was issued, as this is simple to do and serves as a reference case in the subsequent analysis. The tax advantage of debt is denoted by  $\tau$ .<sup>6</sup>

The unlevered value of the firm, for a given closure policy, is readily obtained. If operations are abandoned the first time the state variable  $x_t$  reaches a lower level  $y$ , the after-tax value of the firm is

$$U(x_t | y) \equiv (1 - \tau) \left( \Pi(x_t) + [V^*(y) - \Pi(y)] \mathcal{P}(x_t \triangleright y) \right). \quad (2)$$

The first term on the right hand side,  $(1 - \tau) \Pi(x_t)$ , is the after-tax value of a perpetual entitlement on the current flow of income. The second term is the product of the change in asset value intervening when the irreversible regime switch occurs,  $(1 - \tau) [V^*(y) - \Pi(y)]$ , and a probability-weighted discount factor for this event,  $\mathcal{P}(x_t \triangleright y)$  which we now define.

We assume risk neutrality and a constant safe interest rate,  $\rho$ .<sup>7</sup> We denote by  $T \equiv \inf\{\hat{t} | x_{\hat{t}} = y\}$  the first time at which the state variable  $x_t$  hits the level  $y$ , and by  $f_t(T)$  the density of  $T$  conditional on information at  $t$ . Then the probability-weighted discount factor  $\mathcal{P}(x_t \triangleright y)$  is just the Laplace transform of  $f_t(T)$

$$\mathcal{P}(x_t \triangleright y) = \int_t^\infty e^{-\rho(T-t)} f_t(T) dT. \quad (3)$$

Clearly, the optimal closure policy consists of selecting the abandonment trigger level,  $y$ , in order to maximize  $U(x_t | y)$ . The ex ante *optimal* abandonment trigger level, which we denote  $\tilde{y}$ , must therefore satisfy the first order condition

$$\frac{\partial U(x_t | \tilde{y})}{\partial \tilde{y}} = 0. \quad (4)$$

The existence and uniqueness of the optimal abandonment trigger level  $\tilde{y}$  is then guaranteed by:

**Assumption 1** *At entry (state  $x_0$ ), the option value of triggering liquidation at  $y$ ,*

$$(1 - \tau) [V^*(y) - \Pi(y)] \mathcal{P}(x_0 \triangleright y), \quad (5)$$

*is a strictly concave function in  $y$ , maximized at a trigger level  $\tilde{y}$  strictly smaller than  $x_0$ .*

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<sup>6</sup>As Miller (1977) pointed out, taking into account differences in personal taxation,  $\tau = 1 - (1 - \tau_c)(1 - \tau_V)/(1 - \tau_L)$  where  $\tau_c$  is the corporation tax rate.  $\tau_V$  and  $\tau_L$  are respectively the personal tax rate on equity and debt income. We assume that the firm obtains full tax loss offset. This means that tax carryforwards and -backwards are possible without limit and earn interest at the riskfree rate; moreover, any accrued but unclaimed tax credits can be sold upon abandonment of the firm. This assumption avoids that the value of tax effects depends on the firm history.

<sup>7</sup>Harrison and Kreps (1979) show how to extend the results of the paper to a world without risk-neutrality, by using an equivalent martingale measure.

Note that  $\tilde{y}$  does not depend on the tax rate  $\tau$ . We furthermore assume that the manager has the best use for the assets in the good states, and that the manager's presence is indispensable to unlock the value  $\Pi(x)$ . But in low states of the world it becomes eventually optimal for the firm to abandon and sell these assets. *Figure 1* illustrates this set-up.

This structural model of the firm and its uncertain environment, summarized by  $\{x; \Pi(x); \mu(x); \sigma(x); \rho; \tau; V^*(x)\}$ , is expressed in rather general terms. In Section VI, we will consider a standard parametrization of the model, which will permit to derive closed-form solutions for the securities values and the key variables.

The set-up so far is the same as in Mella-Barral (1999), with the only difference that we consider taxes. This similarity is deliberate since it will allow for a direct comparison of the results, and hence for an analysis of the differences between a firm choosing to finance with private debt and a firm issuing publicly traded debt. We will next also adapt Mella-Barral's model to allow for multiple creditors.

### *C. Debt Financing and Shareholder Opportunism*

The preferential tax treatment of debt is the reason why the manager finds it optimal, at date  $t = 0$ , to issue a certain amount of debt. We will eventually determine the optimal capital structure of the firm, but need first to introduce the available debt instruments. We restrict attention to debt contracts of infinite maturity. Thus, at date  $t = 0$ , the incumbent issues bond contracts,  $\mathcal{D}_0$ , which promise a perpetual flow of coupon payments,  $\delta_0$ , and give the right to trigger liquidation of the firm if the manager defaults on her debt service obligation. If such a default event occurs in state  $y$ , each bond carries liquidation rights whose expected value we denote by  $D_0^*(y)$ . A more detailed specification of the structure of the debt contracts, in particular as to components and allocation of the liquidation rights, is postponed until Section II.E, after the fundamental time-consistency conflicts have been introduced.

We assume that there are  $N$  such bonds issued and that each creditor holds only one bond. The number of creditors  $N$  is so large that each creditor will behave atomistically, and in particular completely neglect his impact on success or failure of a debt restructuring proposal.<sup>8</sup> Out-of-court renegotiation is assumed costless, whereas court-supervised reorganization (Chapter 11 of the US Bankruptcy Code of 1978) is assumed to be costly, hence the manager will always renegotiate out-of-court. We furthermore consider that in liquidation, the Absolute Priority Rule is applied and that shareholders receives nothing.

These are clearly simplifying assumptions: In practice, Chapter 11 is often used in the US and deviations from the Absolute Priority Rule are the rule rather than the exception. Technically, the model could be extended to make court-supervised reorganization eventually desirable and to allow for deviations from the Absolute Priority Rule.<sup>9</sup> These features are

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<sup>8</sup>This is a standard assumption since Grossman and Hart (1980).

<sup>9</sup>For example, following Briys and de Varenne (1997).

not central to the mechanisms developed in this paper, but incorporating them would greatly complicate the presentation of results and intuition.

Concerning the basic conflict of interest between shareholders and bondholders, we assume that the manager exercises residual control rights,<sup>10</sup> i.e. the right to freely decide on the use of the assets as long as they meet their contractual obligations. The final control decision appertaining to the manager is the selection of the abandonment trigger level,  $y$ . The manager is assumed to act in the best interests of shareholders, abstracting from the insider-outsider agency conflict between shareholders and management.

The manager decides in continuous time whether and when to renegotiate the debt contract. She decides on a *sequence* of offers launched to obtain concessions from the creditors. The decisions on the renegotiation offers and the final abandonment decision are interdependent since the total amount of concessions on the debt services determines when the manager will find it optimal to trigger the abandonment decision.

The dispersed creditors (bondholders) act as a non-coordinated group in contract renegotiation, and the manager has the possibility to exploit this *non-cohesiveness* of creditors. She is in a position to make opportunistic take-it-or-leave-it offers to the non-coordinated group of creditors. She enforces every exchange offer with a *strategic default* threat: If the offer is rejected, then the manager is committed to default and walk away. The creditors then react optimally by rejecting or accepting the offer without coordinating their responses.<sup>11</sup> Notice that such a strong shareholder bargaining position is certainly even more characteristic and appealing in the context of dispersed creditors than in the single-creditor case considered in Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997).

Trading of assets occurs continuously in perfect and frictionless markets with no asymmetry of information. The manager therefore maximizes solely the value of equity and acts in a purely opportunistic fashion vis-a-vis the bondholders. Shareholders and creditors anticipate fully the impact of the manager's renegotiation offers on her choice of abandonment trigger level,  $y$ .

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<sup>10</sup>This terminology follows Grossman and Hart (1986) and Hart and Moore (1990).

<sup>11</sup>Thus, the game played at every instant is a hierarchical Stackelberg equilibria. The leader (shareholder) commits to a particular strategy, and the follower (creditors) then react optimally, taking the leader's strategy as given. Technically speaking, we consider a stochastic differential game where strategies are Markov, open loop (state dependent), and perfect state (perfect information). Basar and Olsder (1994) provide an extensive discussion of such games.



## II. Debt Renegotiation with Multiple Creditors

### A. Exchange Offer Strategies and Dilution Threats

We account for shareholders' option to make repeated opportunistic take-it-or-leave-it-offers which exploit the non-cohesiveness of creditors considering that the manager can pursue an *exchange offer strategy* formally defined as follows:

**Definition 1** *An exchange offer strategy,  $\mathbf{s} \equiv \{(\underline{x}_k, n_k, \mathcal{D}_k) \mid k \in \{1; \dots; K\}\}$ , is a collection of sequential debt exchange offers  $(\underline{x}_k, n_k, \mathcal{D}_k)$ : When the state variable reaches the threshold level,  $\underline{x}_k$ , the  $k^{\text{th}}$  offer proposes the  $n_k$  first tendering creditors, to exchange their old debt contract for a new one. The new contract,  $\mathcal{D}_k$ , has the same contractual form than the contract it replaces,  $\mathcal{D}_{k-1}$ .*

An exchange offer is a proposal of a possibly limited number of new debt contracts in exchange for voluntary surrender of old contracts, and an exchange offer strategy is a series of such exchange offers. The manager will choose the exchange offer strategy that maximizes equity value. In each offer, the manager, as the Stackelberg leader, commits to cease debt service payments and have shareholders walk away if the offer is rejected, leaving creditors with no other option than to seize the court and to distribute the liquidation value  $V^*(x)$  according to contractual priority.

The number of exchange offers,  $K$ , is endogenously determined by the game between shareholders and creditors. Notice that the manager cannot ex ante commit to a certain number  $K$ . We will consider that  $K$  is a finite number, but this restriction is without loss of generality since the last offer is well defined, as we show. Therefore, any exchange offer strategy can be represented as a finite sequence.

Now, the size of acceptable coupon reductions is clearly limited by incentive compatibility conditions of the creditors. One important consequence of these incentive compatibility conditions is well-known: Gertner and Scharfstein (1991),<sup>12</sup> among others, have shown that *pari passu* offers (equal seniority) will not be accepted. To see the reason, recall that every debt value can be decomposed into two components: (i) the value of pre-abandonment income rights (debt service payments) and (ii) the value of post-abandonment income rights (liquidation rights). Since a non-tendering creditor can assure himself the initial coupon without any negative consequences, he cannot be made to accept a lower coupon without receiving a higher value of his liquidation right.

Since all liquidation proceeds will belong to the creditors anyway (by virtue of the Absolute Priority Rule), the increase in the residual claim must come at the expense of other creditors. Therefore, a successful exchange offer must threaten to relocate wealth *between*

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<sup>12</sup>See Proposition 1 in Gertner and Scharfstein (1991), p. 1200.

*creditors*, or in other words it must dilute the liquidation rights of holdouts. Notice that dilution threats can only work if there are multiple creditors who cannot coordinate their strategies, since a redistribution of wealth between creditors can only be engineered then.

### B. Engineering Dilution: Covenants and Bondholder Votes

Therefore, we need only consider exchange offer strategies implying *dilution*, i.e. a reduction in the liquidation rights of those creditors who decline the offer. We will next introduce devices which are sufficient conditions to engineer this dilution, and we do so by closely following popular procedures in debt exchange offers.

In practice, covenants in the bond indentures often stand in the way of any alterations in the priority structure; this is in particular the case for subordinated debt claims most vulnerable to dilution threats. Typically, bond indentures require some majority or super-majority of  $m \geq 0.5$  to alter any covenant. The covenant can, however, be removed using an exit consent, also known as a consent solicitation,<sup>13</sup> which ties the right to tender for the new bonds to prior approval of the covenant stripping.

Moreover, popular devices in exchange offers are to ration the number of newly available contracts, i.e. to offer strictly less bonds than there are eligible bonds, and to make the offer *conditional* on a high minimum acceptance rate. The rationing is usually interpreted as a means to make creditors “rush in” to tender. The minimum acceptance rate should increase pressure on the bondholders, to make sure that enough consenting votes will be cast.

Therefore, we consider exchange offer strategies using the following dilution devices:

1. A covenant protects every debt contract,  $\mathcal{D}_0, \mathcal{D}_1, \dots, \mathcal{D}_K$ , from the issuance of secured debt or debt of equal or higher seniority.
2. This covenant can be stripped by a majority of  $m = 0.5$  of the bondholders.
3. The  $k^{\text{th}}$  exchange offer is made conditional on at least  $n_k$  creditors tendering.
4. The number of contracts available for exchange in each round is rationed, that is,  $n_k \leq n_{k-1}$  for all  $k \in \{1; \dots; K\}$ .<sup>14</sup>

The most recent contract carries then a protective covenant against the issuance of secured or senior debt, and this covenant must first be removed before the next offer can be

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<sup>13</sup>Section 316(b) of the US Trust Indenture Act of 1939 requires that each individual bondholder agrees to any change in a core term of a bond issue such as principal amount, interest rate, or maturity. However, protective covenants that limit the firm’s capacity to issue senior debt can be altered through a majority or super-majority vote.

<sup>14</sup>The assumption that the number of available new contracts is shrinking with each exchange offer simplifies the calculations greatly: It allows to analyze the strategy choice of an individual creditor without any strategic spillovers, i.e. the value functions of a creditor for its various options vis-a-vis an exchange offer are independent of the other creditors’ choices.

made. Therefore, the next exchange offer will necessarily be made to the creditors holding the most recently issued contract: They are the only ones protected by a covenant, and without their approval of the covenant stripping, no subsequent offer can be made. Their claims are guaranteed to be the exclusive target of the next offer, since every exchange offer seeks only a single class of debt (Definition 1).<sup>15</sup>

The assumption that the  $k^{\text{th}}$  exchange offer is made conditional on  $n_k$  creditors tendering, is without loss of generality because in the equilibria described below, all creditors will tender. We will make use of the accounting convention  $n_0 = N$ .

### C. Exchange Trigger Points, Regimes and Asset Valuation

It will not be optimal for the manager to trigger new offers unless conditions worsen, so the asset valuation problem will be path-dependent only as far as the minimum state is concerned. Therefore, one additional state variable,  $\check{x}_t$ , is sufficient to keep track of the path-dependence.  $\check{x}_t$  denotes the historical minimum reached by the state variable  $x_t$  since the date the initial debt contracts are issued:

$$\check{x}_t \equiv \inf_{0 \leq \kappa \leq t} \{x_\kappa\}. \quad (6)$$

The time interval between the  $k^{\text{th}}$  and the  $k + 1^{\text{th}}$  exchange offer will be referred to as “regime”  $k$ . Given that these offers are respectively triggered the first time  $x_t$  reaches the levels  $\underline{x}_k$  and  $\underline{x}_{k+1}$ , regime  $k$  corresponds to  $\check{x}_t \in (\underline{x}_{k+1}; \underline{x}_k]$ . Immediately after entry the firm is in regime 0, after the first offer in regime 1, and so on until the last regime  $K$  which is maintained until abandonment.

For all  $\check{x}_t \in (\underline{x}_{k+1}; \underline{x}_k]$ , the value of the shares will be denoted by  $S^{(k)}(x_t)$  where the superscript ( $n$ ) designates the regime  $k$ . The  $K + 1$  regimes give a sufficiently fine information partition for our purposes, and we will use the regimes rather than  $\check{x}_t$  in our notation. After  $K$  debt exchanges are completed, the final decision that the manager will take is the abandonment decision, by repudiating debt contracts when  $x_t$  reaches the abandonment level,  $y$ .

We denote by  $\mathcal{T}_k$  the set of successfully tendering debtholders in the  $k^{\text{th}}$  exchange offer, and by  $\mathcal{H}_k$  the set of debtholders that are being held out (or are holding out) in the  $k^{\text{th}}$  round for the first time.

Notice that creditors in both sets  $\mathcal{T}_k$  and  $\mathcal{H}_k$  have successfully tendered in all previous rounds: They are being offered new contracts in the  $k^{\text{th}}$  round, because the covenant replacement mechanism ensures that the set of creditors who tendered in the previous offer,  $\mathcal{T}_{k-1} = \mathcal{T}_k \cup \mathcal{H}_k$ , have held without interruption the right to strip the debt from its covenant.

The value, in regime  $k$ , of the claim of each debtholder who tendered and succeeded

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<sup>15</sup>We can show that our analysis remains virtually unchanged if we allowed for exchange offer strategies with *simultaneous* offers, i.e. offers that seek in every round one or several of the outstanding debt contracts.

in obtaining the new contract in the most recent offer (the  $k^{\text{th}}$  offer) will be denoted by  $D_{i \in \mathcal{T}_k}^{(k)}(x_t)$ . The value, in regime  $k$ , of the claim of each debtholder who was held out in the most recent offer (hence succeeded in all prior offers) will be denoted by  $D_{i \in \mathcal{H}_k}^{(k)}(x_t)$ . Consequently, the total value of debt outstanding is, in regime  $k$ ,  $\sum_{i=1}^N D_{i \in \{\mathcal{T}_k \cup \mathcal{H}_{j \leq k}\}}^{(k)}(x_t)$ .

After the  $k^{\text{th}}$  offer, the value of the claim of a creditor  $i \in \mathcal{H}_j$ , a creditor held out (or holding out) in the  $j^{\text{th}}$  round, is easily determined: Once he is held out, a creditor's expected residual claim value remains unchanged and equal to  $D_{j-1}^*(y)$ . If shareholders will ultimately abandon in the state  $y$ , then this creditor's claim is worth

$$D_{i \in \mathcal{H}_j}^{(k)}(x_t) = \frac{\delta_{j-1}}{\rho} + \left[ D_{j-1}^*(y) - \frac{\delta_{j-1}}{\rho} \right] \mathcal{P}(x_t \triangleright y) \quad \text{where } j \in \{1, \dots, k\} \quad (7)$$

Here,  $\delta_j$  denotes the new coupon offered to tendering debtholders in the  $j^{\text{th}}$  round. We can also write the value of the  $n_k$  debt contracts holding the dilution preventing covenant, when the  $k+1^{\text{th}}$  offer will be made, the first time  $x_t$  reaches  $\underline{x}_{k+1}$ : At the time of the  $k+1^{\text{th}}$  offer, bondholders will rush in to tender their old contracts, but know that they will succeed in getting the new one with probability  $n_{k+1}/n_k$ , and fail with probability  $(n_k - n_{k+1})/n_k$ . The value of the claim of a *tendering* creditor  $i \in \mathcal{T}_k$  is therefore

$$D_{i \in \mathcal{T}_k}^{(k)}(\underline{x}_{k+1}^+) = \frac{n_k - n_{k+1}}{n_k} D_{i \in \mathcal{H}_{k+1}}^{(k)}(\underline{x}_{k+1}) + \frac{n_{k+1}}{n_k} D_{i \in \mathcal{T}_{k+1}}^{(k)}(\underline{x}_{k+1}) . \quad (8)$$

Therefore, the value of the  $n_k$  debt contracts held by creditors who have always exchanged, *before* the  $k+1^{\text{th}}$  offer occurs can be expressed in the following recursive form

$$D_{i \in \mathcal{T}_k}^{(k)}(x_t) = \frac{\delta_k}{\rho} + \left[ D_{i \in \mathcal{T}_k}^{(k)}(\underline{x}_{k+1}^+) - \frac{\delta_k}{\rho} \right] \mathcal{P}(x_t \triangleright \underline{x}_{k+1}) . \quad (9)$$

#### D. The Role of Guaranteed Liquidation Rights

For the  $k^{\text{th}}$  offer to be accepted, it must be engineered so that the proposed new contract,  $\mathcal{D}_k$ , is more desirable than the current one,  $\mathcal{D}_{k-1}$  at the time of the offer, making tendering debtholders  $i \in \mathcal{T}_k$  better off than holdouts,  $i \in \mathcal{H}_k$ .

Since the problem is recursive in nature, this is less straightforward than it might appear: This incentive-compatibility condition contains value functions which depend on possible subsequent exchange offers. For a bondholder to tender in state  $x_t$ , it must be the case that (i) the value from holding out is smaller than the value from tendering, and (ii) the value from tendering must correctly discount for the bondholder's exposure to more strategic exchange offers in the future. A feasible exchange offer strategy must take this recursive structure of the incentive-compatibility constraints into account. As we show next, this yields considerable cutting power as to the set of feasible exchange offer strategies.

Let us for a moment consider only exchange offer strategies where the expected value of bond liquidation rights does not evolve, i.e.  $D_k^*(x) = D_{k-1}^*(x)$  for some  $k \in \{1, \dots, K\}$ . In

other words the only reward given to tendering creditors is (i) a non-subordinated claim (i.e. claim of the highest priority level) on the proceeds from a liquidation sale and (ii) the right to strip this newly created debt from its protective covenant.

Under such strategies, debtholders held out in earlier rounds are always better off than those held out in later rounds. This is because the former will ultimately have accepted less reductions in coupon than the latter. Therefore, in any regime  $k$ ,

$$D_{i \in \mathcal{H}_j}^{(k)}(x_t) < D_{i \in \mathcal{H}_l}^{(k)}(x_t) \quad \text{for all } j > l, \quad \text{where } j \text{ and } l \in \{1, \dots, k-1\}. \quad (10)$$

In this context, repeated offers suffer from a time consistency problem: Debtholders always reject a first exchange offer, because, if the manager has later the possibility to make a second offer, then holding out in the first offer is the only way for creditors to protect against further expropriation. The repeated nature of the problem imposes an important *credibility constraint* on feasible strategies of shareholders, which will ultimately enhance the ex-ante borrowing ability of the firm and provide a justification for the usage of dispersed (public) debt.

Creditors will not tender in a first offer if the total expected liquidation rights value strategically handed out to the creditors tendering in a second offer can be as high as the expected value of liquidation rights of the bonds held by targeted creditors in the first offer. The manager has to refine her offer and to give tendering bondholders *more* than just seniority (which was sufficient in the static world of Gertner and Scharfstein (1991)): She must be able to *commit* that the rewards cannot be diluted again in subsequent offers.

**Lemma 1** *An exchange offer must give tendering creditors an increased amount of liquidation rights which are immune to subsequent dilution threats.*

*Proof:* A proof is given in the Appendix.

Recall that the number of offers,  $K$ , is endogenous and that the manager can always propose yet another offer. Therefore, as long as the liquidation rights are not secure, the manager can and will launch a subsequent offer which expropriates the liquidation rights through the attribution of more senior claims.

According to Lemma 1, the manager must provide a guarantee that the value gain in residual claims of tendering creditors cannot be fully expropriated in subsequent renegotiation rounds. Any such guarantee must set some liquidation rights aside and exclude them from further dilution. In the following, we call *guaranteed liquidation rights* all devices that offer such a credible commitment. Guaranteed liquidation rights correspond notably to collateral, but, if a part of the assets cannot be collateralized, also to debt in the highest class of seniority as determined by the applicable Bankruptcy Code. This will be further discussed in Section VII.

### E. Structure of Debt Contracts

We are now in a position to finally clarify the structure of the debt contracts. In Section II.C, we had introduced the liquidation rights in deliberately vague terms, saying merely that each bond would give rise to an expected liquidation value of  $D_0^*(x)$  if default occurs in state  $x$ . Lemma 1 provides the crucial insight that this expected liquidation value must involve guaranteed liquidation rights. Let  $G_k^*(x)$  denote the guaranteed liquidation rights attached to the debt contract offered in the  $k^{\text{th}}$  exchange offer. We redefine the components of available debt contracts, which we denote  $\mathcal{D}_k \equiv \{\delta_k; G_k^*(x)\}$  where  $k \in \{0, \dots, K\}$ , in the following form:

1. A promise of a perpetual flow of coupon payments,  $\delta_k$ .
2. The right, if the manager repudiates the contract, to impose a prespecified sharing of the liquidation proceeds  $V^*(y)$  (invoking debt collection law). The details of this sharing rule are as follows:
  - (a) A portion  $G_k^*(x)$  of the proceeds of the liquidation sale,  $V^*(x)$ , is guaranteed.
  - (b) Each debtholder is entitled to a par value,  $P = \delta_k/\rho$ , before shareholders receive anything. The proceeds of the liquidation sale which are not guaranteed are distributed according to the Absolute Priority Rule.

Exchange offer strategies of the kind analyzed here can be viewed as transfers from pre-default income rights to increased liquidation rights. Our analysis therefore applies to restructuring package offering this combination in order to overcome the hold-out effect. The repeated nature of possible dilution threats essentially implies that they can only be successful if accompanied by a (credible) pledge that part of the newly extended liquidation right is *irreversible*.

## III. Ex Post Optimal Exchange Offer Strategy

In this section we study the *ex post* behavior of the manager, acting in the best interests of shareholders, that is, we examine the optimal opportunistic exchange offer strategy she can implement, *once  $N$  given debt contracts of the form  $\mathcal{D}_0 = \{\delta_0; G_0^*(x)\}$  are issued.*

### A. The Manager's Optimization Problem

When solving for the shareholders' ex post optimal exchange offer strategy,  $\mathbf{s}$ , the manager works backwards in time, evaluating the entire sequence of decisions available to her, from the final abandonment to the point of entry. Therefore, the manager's ex post optimization problem will be broken down into a recursive sequence of constrained optimization problems. The objective function in the  $k^{\text{th}}$  regime is the equity value  $S^{(k)}(x_t | y)$ , which can be obtained from the firm value of the levered after subtracting the value of all debt claims. The value

of the levered firm corresponds to the value of the unlevered firm plus the value of the debt tax shield,<sup>16</sup>

$$U(x_t | y) + \tau \sum_{i=1}^N D_{i \in \{\mathcal{T}_k \cup \mathcal{H}_{j \leq k}\}}^{(k)}(x_t). \quad (11)$$

Subtraction of all debt claims in the  $k^{\text{th}}$  regime yields the equity value,

$$S^{(k)}(x_t | y) = U(x_t | y) - (1 - \tau) \sum_{i=1}^N D_{i \in \{\mathcal{T}_k \cup \mathcal{H}_{j \leq k}\}}^{(k)}(x_t). \quad (12)$$

For any given exchange offer strategy,  $\mathbf{s} = \{(\underline{x}_k, n_k, \mathcal{D}_k) \mid k \in \{1; \dots; K\}\}$ , the manager first calculates the optimal abandonment trigger level,  $y_s$ , which occurs *after all* exchange offers have been played out. This trigger level  $y_s$ , solves

$$y_s \equiv \arg \max_y S^{(K)}(x_t | y). \quad (13)$$

Proceeding backwards, the manager then calculates the sequence of optimal offers, from the last exchange offer to the point of entry. She optimizes recursively each one of the  $K$  offers, for a *given prior* exchange offer strategy. She does this for all  $k \in \{1; \dots; K\}$ , starting at  $k = K$  and finishing at  $k = 1$ . The result of previous optimizations  $k \in \{j + 1; \dots; K\}$  are fed back into the  $j^{\text{th}}$  exchange offer optimization problem.

The characteristic parameters,  $(\underline{x}_k, n_k, \mathcal{D}_k)$ , of the shareholders' *optimal*  $k^{\text{th}}$  exchange offer, maximize the value of the equity in regime  $k - 1$ ,

$$\max_{\underline{x}_k, n_k, \mathcal{D}_k} S^{(k-1)}(x_t | y_s), \quad (14)$$

$$\text{subject to: } n_{k-1} \geq n_k \geq 1, \quad (15)$$

$$D_{i \in \mathcal{T}_k}^{(k)}(\underline{x}_k) \geq D_{i \in \mathcal{H}_k}^{(k)}(\underline{x}_k) \quad (16)$$

$$\underline{x}_k \leq \underline{x}_{k-1}. \quad (17)$$

Equation (15) is called the “ $k^{\text{th}}$  rationing constraint”, as it reflects the condition that the number of new contracts will be (weakly) lower to the number of contracts previously holding the dilution preventing covenant.

Equation (16) is called the “ $k^{\text{th}}$  tendering constraint”, guaranteeing that tendering the old debt contract is better than holding out. Equation (17) simply insures that  $k^{\text{th}}$  offer is made after the  $k - 1^{\text{th}}$ .

### B. Satisfying the Tendering Condition

In the  $k^{\text{th}}$  exchange offer, as shown in Lemma 1, a commitment against further dilution consists of increased guaranteed liquidation rights,  $G_k^*(x)$ , replacing the old ones,  $G_{k-1}^*(x)$ ,

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<sup>16</sup>The debt tax shield  $\tau \sum_{i=1}^N D_{i \in \{\mathcal{T}_k \cup \mathcal{H}_{j \leq k}\}}^{(k)}(x_t)$  consists of the coupon right component as well as of the liquidation value component of the debt value. This follows from our assumption of full loss offset.

for each tendering creditor. Even if held out in future renegotiations, each tendering creditor is then assured to receive at least  $G_k^*(x)$ , if abandonment occurs in state  $x$ . We can now also clarify how the number of exchange offers,  $K$ , is determined. The last or  $K^{th}$  exchange offer is the offer where the last part of the liquidation rights are fully guaranteed, i.e. when  $V^*(x) = \sum_{j=1}^K (n_{j-1} - n_j)G_{j-1}^*(x) + n_K G_K^*(x)$ . Subsequent offers will be rejected, according to Lemma 1, and are irrelevant for the equilibrium outcome.

The question is then how much new guaranteed liquidation rights must be added at every round for the exchange offer to be *dynamically incentive-compatible*, i.e. to be acceptable for creditors rationally anticipating that further exchange offers are possible. We find that:

**Lemma 2** *The  $k^{th}$  tendering condition,  $D_{i \in \mathcal{T}_k}^{(k)}(\underline{x}_k) \geq D_{i \in \mathcal{H}_k}^{(k)}(\underline{x}_k)$ , can be written*

$$G_k^*(y_s) - G_{k-1}^*(y_s) \geq \frac{(\delta_{k-1} - \delta_k)}{\rho} \frac{[1 - \mathcal{P}(\underline{x}_k \triangleright y_s)]}{\mathcal{P}(\underline{x}_k \triangleright y_s)}. \quad (18)$$

*Proof:* A proof is given in the Appendix.

Starting from the observation that an exchange must contain an irrevocable pledge of *more* guaranteed liquidation rights (Lemma 1), Lemma 2 quantifies the minimum value of this irreversible additional pledge. Each of the  $K$  consecutive offers must offer sufficient new guaranteed liquidation rights to meet condition (18). After the  $k^{th}$  successful offer, the remaining claims to the liquidation value that the manager can still redistribute strategically in subsequent offers is bounded by the value of not yet guaranteed liquidation rights,  $V^*(x) - \sum_{j=1}^k (n_{j-1} - n_j)G_{j-1}^*(x) - n_k G_k^*(x)$ .

Throughout, we restrict attention to the following equilibrium outcome in each exchange offer: Once the incentive constraint,  $D_{i \in \mathcal{T}_k}^{(k)}(\underline{x}_k) \geq D_{i \in \mathcal{H}_k}^{(k)}(\underline{x}_k)$ , is satisfied, all of the remaining  $n_{k-1}$  creditors tender. This implies that all  $n_k$  new contracts on offer will actually be exchanged. This is a subgame perfect equilibrium outcome since the sequence of dynamic incentive constraints ensures that the creditors' strategies are best responses.

This outcome is typically not unique,<sup>17</sup> but selecting this particular equilibrium can be justified on the grounds that it is the *efficient* equilibrium as long as debt renegotiation adds value to the firm - and this is indeed the case as we will show (Proposition 2). Restricting attention to the efficient equilibrium is a frequently used approach for coordination games like the present one, and game theory provides various arguments justifying this selection.<sup>18</sup>

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<sup>17</sup>For example, if  $n_{k-1} - 1$  creditors reject the offer in the  $k^{th}$  renegotiation round, then rejecting constitutes a (subgame perfect) equilibrium response for the remaining creditor even if the incentive constraint (18) holds strictly and the outcome is independent of the last creditor's response.

<sup>18</sup>In particular, evolutionary game theory concepts (see e.g. Kandori, Mailath and Rob (1993)) or the Harsanyi-Selten equilibrium selection theory.



### C. Optimal Exchange Offers

We can now rewrite the manager's optimization problem (14) - (17), by replacing the  $k^{\text{th}}$  tendering constraint (16) with the more specific condition (18), in terms of the characteristic variables  $(\underline{x}_k, n_k, \delta_k, G_k^*(x))$  that she uses to directly control the exchange offer strategy.

To make further progress, we first establish the following crucial Lemma:

**Lemma 3** *If exchange offer strategy  $s$  is optimal, then the tendering constraint (18) is binding for every exchange offer  $k \in \{1; \dots; K\}$ .*

*Proof:* A proof is given in the Appendix.

This Lemma has a straightforward intuition (though the formal demonstration is involved), since in every exchange offer, reducing the new coupon on offer promises shareholders a twofold gain. First, it reduces the debt service payments value over the expected time horizon until the firm is liquidated. Second, since the abandonment trigger level  $y$  is monotonic in the final aggregate coupon value, it prolongs the life expectancy of the firm, and over the additional life span, the equity value must be positive. Thus, the manager reduces the new coupon on offer until the tendering constraint binds.

The fact that the tendering constraint is binding at every exchange turns out to be powerful in this model: Taking the expression for  $D_{i \in \mathcal{H}_j}^{(k)}(x_t)$  in equation (9), it implies

$$D_{i \in \mathcal{T}_k}^{(k)}(\underline{x}_k) = D_{i \in \mathcal{H}_k}^{(k)}(\underline{x}_k) = \frac{\delta_{k-1}}{\rho} + \left[ G_{k-1}^*(y_s) - \frac{\delta_{k-1}}{\rho} \right] \mathcal{P}(\underline{x}_k \triangleright y_s), \quad (19)$$

for all  $k \in \{1; \dots; K\}$ . In particular, this is true for  $k = 1$ ,

$$D_{i \in \mathcal{T}_1}^{(1)}(\underline{x}_1) = D_{i \in \mathcal{H}_1}^{(1)}(\underline{x}_1) = \frac{\delta_0}{\rho} + \left[ G_0^*(y_s) - \frac{\delta_0}{\rho} \right] \mathcal{P}(\underline{x}_1 \triangleright y_s). \quad (20)$$

Therefore replacing in the value of a bond in the initial regime,

$$D^{(0)}(x_t) = \frac{\delta_0}{\rho} + \left[ D_{i \in \mathcal{T}_1}^{(1)}(\underline{x}_1) - \frac{\delta_0}{\rho} \right] \mathcal{P}(x_t \triangleright \underline{x}_1), \quad (21)$$

$$= \frac{\delta_0}{\rho} + \left[ G_0^*(y_s) - \frac{\delta_0}{\rho} \right] \mathcal{P}(x_t \triangleright y_s). \quad (22)$$

Obviously, once the debt is issued, a bondholder can always decide never to tender, and by doing so, guarantee himself a coupon flow of  $\delta_0$  until operations are abandoned and then the guaranteed liquidation rights  $G_0^*(x)$  initially contracted upon. The debt value cannot possibly be reduced below this reservation value, whatever the manager's dilution efforts. But then note that Equation (22) expresses precisely this reservation value. Thus, we have established that the manager is always able to define a strategy that squeezes creditors to this reservation value. In essence, creditor's initial entitlement to a share of the proceeds from

a liquidation sales which cannot be qualified as guaranteed liquidation rights is *worthless* since it will be expropriated through an exchange offer strategy, before repudiation. Hence we have established that  $D_k^*(x) = G_k^*(x)$ , for all  $k \in \{0, \dots, K\}$ : The ex ante value of non-guaranteed liquidation rights is ex post fully appropriated by the opportunistic shareholder. Solving for the managers' ex post optimal offer strategy, we find that the manager is able to keep the debt value to the *creditor reservation value*.

Replacing equation (22) into equation (12), we can rewrite the equity value as

$$S^{(0)}(x_t | y_s) = U(x_t | y_s) - (1 - \tau) N D^{(0)}(x_t), \quad (23)$$

where the abandonment trigger  $y_s$  is as defined in (13).

## IV. Ex Ante Financing and Contract Design

So far, we studied the ex post behavior of the shareholder, assuming the project to be financed with  $N$  *given* debt contracts  $\mathcal{D}_0 \equiv \{\delta_0; G_0^*(x)\}$ . Working backwards in time, we turn to the question of optimizing the firm's capital structure. Taking the opportunistic ex post optimization into account, we determine which debt contract shareholder and creditors will find feasible and optimal at the date of entry.

### A. Ex Ante Optimization Problem

At the date of entry, the optimal capital structure (the optimal  $N$  debt contracts  $\mathcal{D}_0 \equiv \{\delta_0; G_0^*(x)\}$ ) maximizes the total value of the firm,  $S^{(0)}(x_0 | y_s) + N D^{(0)}(x_0)$ , or

$$\max_{\delta_0, G_0^*(x)} \{ S^{(0)}(x_0 | y_s) + N D^{(0)}(x_0) \} \quad (24)$$

$$\text{subject to: } 0 \leq N G_0^*(x) \leq V^*(x), \quad \text{for all } x \quad (25)$$

$$y_s = \arg \max_y S^{(0)}(x_0 | y). \quad (26)$$

The choice of the function of initially guaranteed liquidation rights,  $G_0^*(x)$ , must be feasible. This requires that (i)  $G_0^*(x) \geq 0$ , since creditors are protected by limited liability and hence their guaranteed payoff in case of liquidation cannot be negative; and (ii) the guarantees cannot pledge more than the total liquidation value available,  $G_0^*(x) \leq V^*(x)/N$ . Within these limits, we assume that the functional form  $G_0^*(x)$  can be freely chosen so as to maximize the ex ante value. Without loss of generality, we assume that the manager chooses a differentiable function  $G_0^*(x)$ .<sup>19</sup>

The manager is constrained to choose a path that she is able to follow through ex post, i.e. a path that is time consistent at any moment. We show in the Appendix, Result 1, that

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<sup>19</sup>Typically, the value of the guaranteed liquidation rights will evolve according to the value of the assets assigned as collateral. In essence, we assume that either enough flexibility exists in the choice of a subset among the firm's assets that is given as collateral, or that financial engineering permits to circumvent any restrictions imposed by the firm's assets.

once the manager has identified her ex ante preferred abandonment point, she will be able to adopt a strategy that ensures she will stick to this abandonment point after every possible future path. The initially optimal policy is thus time-consistent.

Ex ante, the manager seeks to choose the two instruments of the debt contract,  $\delta_0$  and  $G_0^*(x_t)$  in such a way as to maximize the ex ante value,  $S(x_0 | y_s) + ND^{(0)}(x_0)$ , by taking into account that her abandonment decision  $y_s$  will be a function of  $\delta_0$  and of  $G_0^*(x_t)$ . In particular, this maximization implies that

$$\frac{d}{d\delta_0} (S(x_0 | y_s) + ND^{(0)}(x_0)) = 0 . \quad (27)$$

After a few manipulations, condition (27) allows for the following insight.

**Lemma 4** *The manager will raise the initial coupon  $\delta_0$  until the marginal tax benefit is just equal to the marginal loss from premature abandonment,*

$$\tau \frac{\partial ND^{(0)}(x_0)}{\partial \delta_0} = - \frac{1}{(1 - \tau)} \frac{\partial U(x_0 | y_s)}{\partial y_s} \frac{\partial y_s}{\partial \delta_0} . \quad (28)$$

*Proof:* A proof is given in the Appendix.

Thus, the manager's ex ante capital structure choice is characterized by the familiar "static trade-off" between the tax advantage of leverage and increasing expected bankruptcy costs - here represented by early abandonment of the firm - that is a staple of corporate finance and that is also at the heart of Leland's (1994) model. As in Leland's case, in the absence of taxes ( $\tau = 0$ ), condition (28) implies abandonment at the first best,  $y_s = \tilde{y}$ , whereas for any positive tax rate  $\tau > 0$ , we have  $y_s > \tilde{y}$ . Here, the fact that debt renegotiation is possible does not change fundamentally this ex-ante trade-off.

For a given abandonment point  $y$ , the larger the debt value  $ND^{(0)}(x_0)$ , the higher will be the shareholders' ex ante value  $U(x_0 | y) + \tau ND^{(0)}(x_0)$ . This is a direct consequence of the tax advantage of debt. The policy that maximizes the initial debt value, among all the capital structure policies leading to the same abandonment decision  $y$ , will be ex ante optimal.

Therefore, we proceed by determining the minimal ex ante equity level that is compatible with the manager pursuing ex post an optimal exchange offer strategy and abandoning at a given  $y$ . One possible alternative strategy for the manager is easily identified: the manager has ex post always the possibility to never renegotiate. In this case, her ex post optimal abandonment level is a function of the initial coupon  $\delta_0$  alone. We will denote this optimal abandonment level by  $y_f(\delta_0)$ , and the associated equity value by  $S_f^{(0)}(x_0 | y_f)$ .

Obviously, the manager will only adopt the optimal exchange offer strategy  $\mathbf{s}$  if the resulting equity value  $S^{(0)}(x_0 | y_s)$  is at least as large than her value under this alternative

strategy,  $S_f^{(0)}(x_0 | y_f)$ . In equilibrium, this incentive constraint will indeed be binding (see the Appendix):

$$S^{(0)}(x_0 | y_s) = S_f^{(0)}(x_0 | y_f) . \quad (29)$$

### B. Optimal Debt Contract

We will next have a closer look at the optimal balance between the two instruments of the debt contract, the coupon  $\delta_0$  and the guaranteed liquidation rights  $G_0^*(x_t)$ . The larger the coupon  $\delta_0$ , the earlier will the manager abandon ex post; likewise, the larger is the terminal value of guaranteed liquidation rights  $G_0^*(x_t)$ , the less concessions can be obtained and hence the earlier will be abandonment. Thus, there is a range of combinations  $(\delta_0, G_0^*(x_t))$  that will give rise to the same abandonment point  $y$ , and the two instruments are substitutes over this range. Following the argument developed earlier, the optimal point within the range of combinations leading to the same abandonment decision  $y$  will be the combination that gives rise to the highest ex ante debt value  $D^{(0)}(x_0)$ . Finding this optimum, however, is not immediately obvious since the initial debt value is monotonically increasing in both the coupon value and the value of guaranteed liquidation rights.

The optimum is in fact characterized by a corner solution. Namely, as we compare different combinations  $(\delta_0, G_0^*(x_t))$  for a given  $y$ , we find that the debt value is always rising more in the coupon increase than it is falling in the corresponding reduction in  $G_0^*(y)$ , needed to keep  $y$  constant. Hence, in an optimal debt contract,

$$G_0^*(y_s) = 0 . \quad (30)$$

A proof of equation (30) is given in the Appendix. Note that this insight pins down the value of guaranteed liquidation rights only for the abandonment point  $y_s$ . In general,  $G_0^*(x) > 0$  for all  $x > y_s$  will be required, as we will discuss shortly.

We summarize our results rewriting the characteristic equations (28), (29), (30) and (26) in terms of the inputs of the model, and restating the valuation equations (22) and (23):

**Proposition 1** *The ex-ante optimal debt contract,  $\mathcal{D}_0 \equiv \{\delta_0; G_0^*(x)\}$ , is characterized by*

$$N \frac{\tau}{\rho} [1 - \mathcal{P}(x_0 \triangleright y_s)] + \frac{\partial ( [V^*(y_s) - \Pi(y_s)] \mathcal{P}(x_0 \triangleright y_s) )}{\partial y_s} \frac{\partial y_s}{\partial \delta_0} = 0 , \quad (31)$$

$$\delta_0 = \frac{\rho}{N [1 - \mathcal{P}(y_f \triangleright y_s)]} \left[ \Pi(y_f) + (V^*(y_s) - \Pi(y_s)) \mathcal{P}(y_f \triangleright y_s) \right] , \quad (32)$$

$$G_0^*(y_s) = 0 , \quad (33)$$

$$\frac{\partial}{\partial y_s} \left( [V^*(y_s) - \Pi(y_s) - N G_0^*(y_s) + N \delta_0 / \rho] \mathcal{P}(x_0 \triangleright y_s) \right) = 0 . \quad (34)$$

*These four equations determine the coupon  $\delta_0$ , the value and slope of the guaranteed liquidation rights at abandonment,  $G_0^*(y_s)$  and  $dG_0^*(y_s)/dy_s$ , and  $y_s$ .  $y_f$  corresponds to  $y_s$  for  $G_0^*(x) = V^*(x)/N$ .*

This contract induces the manager to follow a non-cooperative optimal exchange offer strategy  $\mathbf{s}$  such that the first renegotiation takes place after the first time  $y_f$  is reached and abandonment ultimately occurs at  $y_s > \tilde{y}$ .

The associated values of each bond and the equity are, respectively, for all  $x_t > y_f$ ,

$$D^{(0)}(x_t) = \frac{\delta_0}{\rho} [1 - \mathcal{P}(x_t \triangleright y_s)] , \quad (35)$$

$$S^{(0)}(x_t | y_s) = U(x_t | y_s) - (1 - \tau) N D^{(0)}(x_t) . \quad (36)$$

*Proof:* A proof is given in the Appendix.

Note that once we add a standard parameter structure for the values  $\Pi(x_t)$ ,  $V^*(x_t)$  and the underlying process  $x_t$  (see Section VI.), the characterization of the optimal debt contract in Proposition 1 will yield closed form expressions for the value of assets.

In economic terms, the conditions (31) - (33) reflect the following considerations determining the optimal choice of the instruments  $(\delta_0, G_0^*(x))$ . On the one hand, the *option to renegotiate* when the firm approaches the lower reorganization bound (low  $x_t$ ) allows to increase the debt tax shield and hence the firm value. On the other hand, creditors must be given *protection from the premature exercise* of the imbedded debt renegotiation options, and this is achieved through a judicious choice of the slope of  $G_0^*(x)$ . The slope of  $G_0^*(x)$  must be sufficiently steep so as to reward the manager for being patient in proposing exchange offers. That is, the longer she waits, and the lower is therefore  $y$ , the more valuable must be her effective bargaining chip,  $V^*(y) - N G_0^*(y)$ . We have already seen that  $G_0^*(y_s) = 0$ . Hence, since  $V^*(x) \geq N G_0^*(x)$  for all  $x$ , the lower the final abandonment point  $y$ , the larger must be  $N G_0^*(x)$  initially, i.e. at the earliest possible abandonment point.

As a consequence, renegotiable debt with multiple creditors imposes conditions on the design of the function of guaranteed liquidation rights,  $G_0^*(x)$ , that can be illustrated as follows. Consider the extreme case where the initial contract does *never* contain any guaranteed liquidation rights, i.e. the initial debt contract  $\mathcal{D}_0 \equiv \{\delta_0; G_0^*(x)\}$  involves  $G_0^*(x) = 0$  for all  $x$ . If  $G_0^*(x) = 0$  for all  $x$ , the shareholder's optimal strategy would be to make a single exchange offer leading to full guaranteeing of the debt's liquidation rights *immediately* at the date of entry,  $x_0$ , irrespective of the coupon  $\delta_0$ . The same holds if the shareholder were to issue guaranteed liquidation rights with a value evolution *proportional* to the liquidation value of the firm's assets,  $N G_0^*(x) = \alpha V^*(x)$  for some constant  $\alpha$ .

In either case, the option value of contingent renegotiation. Intuitively, this value arises from the possibility to maintain a high coupon value as long as the firm is able to afford it, and to deploy the renegotiation option in a contingent manner when the firm falls on hard times.

## V. The Value of Renegotiable Debt and of Debt Dispersion

Having characterized the optimal debt contract, we turn to a closer investigation of the two important characteristics of the type of debt considered in this paper: the fact that debt is raised from many creditors, and that it is renegotiable. We compare the debt contract characterized in Proposition 1 with non-renegotiable debt on the one hand and debt held by a single creditor on the other hand.

### A. Non-Renegotiable Debt

Consider the case where the debt's liquidation right is *fully* guaranteed, i.e. the initial debt contract  $\mathcal{D}_0 = \{\delta_0; G_0^*(x)\}$  involves  $G_0^*(x) = V^*(x)/N$  for all  $x$ . This extreme case allows us to derive the following important insight follows directly from Lemma 1:

**Corollary 1** *Debt with fully guaranteed liquidation rights cannot be renegotiated.*

Corollary 1 sheds light on a prominent special case in the structural pricing literature, the valuation of *non-renegotiable* debt claims, as they are assumed in Merton (1974), Leland (1994) and Leland and Toft (1996) in particular. In other words, our model can explain the two joint conditions which make the assumption of non-renegotiability realistic: (i) debt claims are widely dispersed and (ii) shareholders have no latitude to make dilution threats. The latter is true when all liquidation rights are guaranteed.

For this special case, since there will be no renegotiation, the optimal abandonment point can be directly determined from the initial debt contract  $\mathcal{D}_0 = (\delta_0; G_0^*(x))$ . We will denote by  $y_f$  the shareholders' optimal abandonment trigger level in this case, i.e.  $y_f$  is  $y_s$  for the special case of *fully* guaranteed debt.<sup>20</sup> We can then immediately derive the values of each bond and the equity, which for clarity we will denote  $D_f^{(0)}(x_t)$  and  $S_f^{(0)}(x_t)$ , respectively.

$$D_f^{(0)}(x_t) = \frac{\delta_0}{\rho} + \left[ \frac{V^*(y_f)}{N} - \frac{\delta_0}{\rho} \right] \mathcal{P}(x_t \triangleright y_f), \quad (37)$$

$$S_f^{(0)}(x_t) = (1 - \tau) \left[ \Pi(x_t) - \frac{\delta_0}{\rho} - \left( \Pi(y_f) - \frac{\delta_0}{\rho} \right) \mathcal{P}(x_t \triangleright y_f) \right]. \quad (38)$$

Obviously, the abandonment point  $y_f$  is identical to the abandonment point if the manager voluntarily does not renegotiate, an option we had already discussed in Section IV. (cf. Proposition 1). It is a function of the initial coupon only,  $y_f(\delta_0)$ , since the level of initial guarantees is by definition fixed at its maximum. Now an increase in the coupon obligation precipitates shareholder's abandonment, hence increases  $y_f$ . That is, since  $-N[1 - \mathcal{P}(x_t \triangleright y)]\delta_0/\rho$  is negative and strictly increasing in  $y$ , for all  $y < x_t$ , Assumption 1 implies  $\partial y_f(\delta_0)/\partial \delta_0 > 0$ .

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<sup>20</sup>We already introduced the notation  $y_s$  etc. for the isomorphic case where the manager voluntarily does not renegotiate.

To summarize, for debt contracts where  $N G_0^*(x) = V^*(x)$ , for all  $x \leq x_0$ , i.e., for fully collateralized debt, we know that debt will never be renegotiated (Corollary 1), and the final abandonment point will be at  $y_f$ .

### B. The Role of Debt Renegotiability

We can establish the following key insight on the value of having renegotiable debt:

**Proposition 2** *It is always possible to achieve a larger ex ante company value by issuing renegotiable debt compared with the optimal issue of fully collateralized debt.*

*Proof:* A proof is given in the Appendix.

Thus, the capacity to renegotiate debt ex post is valuable, because it allows to exploit the tax advantage of debt without incurring large losses from premature liquidation. The renegotiation option adds value on both sides of the capital structure trade-off: First, creditor concessions imply that the abandonment point  $y_s$  will be closer to the efficient point,  $\tilde{y}$ . Second, it allows to raise more debt initially, which is especially valuable during good times.

It is actually possible to show that (i) the ex ante optimal debt value and (ii) the initial coupon level are actually both always larger with renegotiable debt compared with non-renegotiable debt.

### C. Single Creditor Debt

Our next question is whether the manager should borrow from a single creditor, who efficiently internalizes the value created in every renegotiation round, or whether she should borrow from dispersed creditor whose non-cohesiveness can be exploited. To better understand the relationship between creditor dispersion and debt capacity, we have to compare to the case where there is just a single creditor, i.e.  $N = 1$ . We consider, as we have considered so far, that the manager is in a position to make opportunistic offers to the creditor.

This case has been studied in Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997), and Mella-Barral (1999).<sup>21</sup> When there is a *single* creditor (holding all  $N$  bonds), and the manager is in a position to make strategic default threats (take-it-or-leave-it offers after defaulting) to the creditor, the debt is first renegotiated at a certain threshold level,  $x_s$ . In our setting, which incorporates taxes in Mella-Barral (1999), the values of each bond and the equity are

$$D_s(x_t) \equiv \frac{\delta_0}{\rho} + \left[ \frac{V^*(x_s)}{N} - \frac{\delta_0}{\rho} \right] \mathcal{P}(x_t \triangleright x_s) \quad (39)$$

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<sup>21</sup>In Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997), concessions consist of temporary debt service holidays. The shareholder, as the Stackelberg leader, makes take-it-or-leave-it offers to her creditor, strategically paying less than the originally contracted coupon. In Mella-Barral (1999), the shareholder asks for permanent reductions of debt obligations, forcing her creditor repeatedly to forgive part of her debt. In all models, the (blackmailed) creditor will have to accept any concession giving him a new debt value of exactly  $V^*(x)$ , his outside option.

$$S_s(x_t) \equiv U(x_t | \tilde{y}) - (1 - \tau) N D_s(x_t) , \quad (40)$$

where  $x_s$  solves  $\partial D_s(x_s)/\partial x_s = 0$  and shareholders' ex post optimal abandonment point

$$y = \arg \max (1 - \tau) \left\{ \left( \Pi(x_t) + [V^*(y) - \Pi(y)] \mathcal{P}(x_t \triangleright y) \right) - N D_s(x_0) \right\} = \tilde{y} . \quad (41)$$

#### D. The Role of Creditor Dispersion

Creditor dispersion has a drastic effect on the *debt capacity* of the firm. This is the absolute limit to the amount creditors are willing to lend, or the highest feasible aggregate value of bonds issued at the entry point  $x_0$ . Denote by  $\Lambda(x_0)$  and  $\Lambda_s(x_0)$  the debt capacity of the firm (a) with dispersed creditors and (b) with a single creditor, respectively

$$\Lambda(x_0) \equiv N \max_{\delta_0, G_0^*(x)} \{ D^{(0)}(x_0) \} , \text{ and } \Lambda_s(x_0) \equiv N \max_{\delta_0} \{ D_s(x_0) \} . \quad (42)$$

Now, because of the presence of the strategic default threat, the absolute limit to the amount a single creditor is willing to lend at entry,  $\Lambda_s(x_0) = N \max D_s^{(0)}(x_0)$ , is exactly equal to the liquidation value of the firm,  $\Lambda_s(x_0) = V^*(x_0)$ .<sup>22</sup> We then obtain:

**Lemma 5** *The debt capacity with dispersed creditors,  $\Lambda(x_0)$ , is always strictly larger than the debt capacity with a single creditor,  $\Lambda_s(x_0)$ .*

*Proof:* A proof is given in the Appendix.

The reason for the larger debt capacity<sup>23</sup> is that with dispersed creditors, the strategic default threat does not work: recall that any individual creditor is so small that her acceptance/rejection decision is not decisive for the outcome. Hence, if all other creditors were to accept the offer reducing their aggregate value to  $V^*(x)$ , the best strategy for an individual creditor would be to hold out. With dispersed debtholders, strategic default threats do not improve the bargaining position of the manager, and her only way to get concessions is to pledge additional guaranteed liquidation rights as described earlier.

Based on this insight, we will next explore the implications of our model for the choice between concentrated and dispersed debt. There are in fact two differences between single-creditor debt and dispersed debt that constitute the determinants of this choice. First, when facing a single creditor, the debt capacity is limited by the expectation that the manager can default strategically, whereas dispersed debt amounts to a credible commitment that strategic default threats cannot be used. Second, with a single creditor, the ultimate abandonment decision will always be at the ex post efficient point  $\tilde{y}$ , whereas with dispersed creditors, abandonment will be premature (Lemma 4).

<sup>22</sup>See Equation (44), Section 6.1., of Mella-Barral (1999).

<sup>23</sup>Berglöf, Roland and von Thadden (2000) come to similar conclusions about debt capacity and creditor dispersion in an incomplete contracts model.



With a single creditor, any feasible increase in the debt level does not lead to a higher liquidation loss, since abandonment will remain fixed at the efficient point  $\tilde{y}$ . Therefore, the unique optimal capital structure is to push the initial debt level up to the maximum feasible level,  $ND_s^{(0)}(x_0) = V^*(x_0)$ . By issuing widely dispersed debt, the manager is able to borrow more than by borrowing from just one lender, but she faces the prospect of inefficient abandonment. Thus, the trade-off between the larger debt capacity of dispersed debt and the lower expected liquidation loss with a single creditor determines which of the two is preferable:

**Proposition 3** *It is optimal to issue dispersed debt for low initial liquidation values  $V^*(x_0)$ , and single-creditor debt for high values of  $V^*(x_0)$ .*

*Proof:* A proof is given in the Appendix.

## VI. Implementing the Model

In this Section, we add conditions under which closed-form solutions can be obtained for all the concepts and results of the paper. The closed-form solution allows for a *quantitative* appraisal of the effects presented here, and to examine the importance of creditor dispersion and guaranteed liquidation rights dimensions for the valuation of debt claims.

### A. Closed-Form Solutions

To obtain closed-form solutions, additional structural assumptions are required in order to (i) express the Laplace transform,  $\mathcal{P}(x_t \triangleright y)$ , in simple fashion and to (ii) solve explicitly for the different optimal decision trigger levels, using the relevant first order optimality conditions. We propose a structure, namely Geometric Brownian Motion plus linear income processes, which is reasonably general<sup>24</sup> and simple. There also exist alternative model specifications allowing to implement closed-form solutions.

**Assumption 2 (GBM-Linear Structure)** : (i) *The uncertain state variable,  $x_t$ , describing the current status of the firm follows a geometric Brownian motion,*

$$dx_t = \mu x_t dt + \sigma x_t dB_t, \quad (43)$$

*where  $\mu < \rho$  and  $\sigma$  are constants, and  $B_t$  is a standard Brownian motion.*

(ii) *The value of the firm's operations income flow,  $\Pi(x_t)$ , and the liquidation value of the*

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<sup>24</sup>This structure actually encompasses that of many existing corporate debt valuation models, including Merton (1974), Black and Cox (1976), Brennan and Schwartz (1984), Fischer, Heinkel and Zechner (1989), Mello and Parsons (1992), Kim, Ramaswamy and Sundaresan (1993), Longstaff and Schwartz (1995), Leland (1994), Leland and Toft (1996), Fries, Miller and Perraudin (1997) and Mella-Barral and Perraudin (1997). They either take the total value of the firm's assets or the price of the commodity produced as the driving process, and all assume  $x_t$  to follow a geometric Brownian motion.

firm,  $V^*(x_t)$ , are linear functions of the uncertain state variable,

$$\Pi(x_t) = \Theta_0 + \Theta_1 x_t, \quad \text{and} \quad V^*(x_t) = \Theta_0^* + \Theta_1^* x_t, \quad (44)$$

where the constants  $\Theta_0, \Theta_1, \Theta_0^*$ , and  $\Theta_1^*$ , are such that  $\Theta_0 < \Theta_0^*$  and  $\Theta_1 > \Theta_1^*$ .<sup>25</sup>

Under Assumption 2,  $\mathcal{P}(x \triangleright y)$  can be expressed as

$$\mathcal{P}(x \triangleright y) = \left(\frac{x}{y}\right)^\lambda, \quad \text{where} \quad \lambda \equiv \sigma^{-2}[-(\mu - \sigma^2/2) - ((\mu - \sigma^2/2)^2 + 2\rho\sigma^2)^{1/2}]. \quad (45)$$

Secondly, all asset pricing formulas have a simple functional form

$$A(x_t) = a_A + b_A x_t + c_A x_t^\lambda, \quad \text{where} \quad (a_A, b_A, c_A) \in \mathbf{R}^3, \quad (46)$$

for  $A(x_t) \in \{S^{(0)}(x_t); D^{(0)}(x_t); S_f^{(0)}(x_t); D_f^{(0)}(x_t); S_s(x_t); D_s(x_t); V(x_t | y)\}$ . Solving for the decision trigger levels also yields simple expressions.

Table 1 contains the explicit expressions of the constants  $(a_A, b_A, c_A)$  for all asset  $A(x_t)$ , as well as the abandonment points of (a) the unlevered firm,  $\tilde{y}$ , (b) the levered firm with non-renegotiable debt,  $y_f$ , (c) the first renegotiation point for a levered firm with diffusely held debt,  $y_f(\delta_0)$ , and (d) the first renegotiation point with single-creditor debt,  $x_s$ .

Finally, the characteristics of the ex-ante optimal contract with dispersed creditors given by equations (32), (31), (33) and (31) become respectively

$$\delta_0 = \frac{\rho}{N [1 - (y_f/y_s)^\lambda]} \left[ \Theta_0 + \Theta_1 y_f + \left[ \Theta_0^* - \Theta_0 + (\Theta_1^* - \Theta_1) y_s \right] \left(\frac{y_f}{y_s}\right)^\lambda \right], \quad (47)$$

$$y_s = \frac{1}{(1 - \lambda)(\Theta_1 - \Theta_1^*)} \left( -\lambda(\Theta_0^* - \Theta_0) + \tau \left[ \left(\frac{y_s}{x_0}\right)^\lambda - 1 \right] \left[ N \frac{\delta_0}{\rho} + \Theta_0^* - \Theta_0 \right] \right) \quad (48)$$

$$G_0^*(y_s) = 0, \quad (49)$$

$$\frac{dG_0^*(y_s)}{dy_s} = \frac{1}{N y_s} \left[ (1 - \lambda)(\Theta_1^* - \Theta_1) y_s - \lambda(\Theta_0^* - \Theta_0 + N\delta_0/\rho) \right]. \quad (50)$$

### B. Model Specification and its Impact on Capital Structure and Debt Pricing

The practitioner's question will be: does the model specification matter? We will therefore examine the impact of (i) creditor dispersion and (ii) the guaranteeing of the debt's liquidation rights on the optimal capital structure of the firm and on the security prices. We will then proceed to evaluate the potential error due to a misspecification of the debt type, to see whether the prediction errors for the capital structure and credit spreads are economically relevant.

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<sup>25</sup>Notice that Assumption 1 is satisfied under these conditions.

To investigate the importance of a correct specification of the debt model, we implement the comparison introduced in Section V., between our model and (i) the non-renegotiable debt model (debt with fully guaranteed liquidation rights), tantamount to an adaptation of the Leland (1994) model and (ii) the single-creditor debt model, tantamount to the Mella-Barral (1999) model.

For each type of debt, we determine the ex-ante optimal debt contract issued at a common entry point,  $x_0$ . This immediately gives us the optimal capital structure of the firm and its total value. We then calculate the resulting differences in credit spreads and associated risk premium under these respectively optimal capital structures.

We present a simple numerical application of the closed-form pricing formulas derived under the “GBM-Linear” structure in order to gain a quantitative appraisal of the impact of the debt model specification.

**Input Parameters:** *The entry state is  $x_0 = 30$ . The income uncertainty process,  $x_t$ , fluctuates with  $\mu = 3\%$  and  $\sigma = 30\%$ . The value of the firm’s operations income flow,  $\Pi(x_t) = \Theta_0 + \Theta_1 x_t$ , is such that  $\Theta_0 = -4$  and  $\Theta_1 = 1$ . The liquidation value of the firm,  $V^*(x_t) = \Theta_0^* + \Theta_1^* x_t$ , is such that  $\Theta_0^* = 4$  and  $\Theta_1^* = 0.1$ . The interest rate is  $\rho = 6\%$ . The effective tax advantage of debt is  $\tau = 13\%$ .<sup>26</sup>*

These input parameters correspond to a firm with a substantial operating rent at the entry point,  $x_0 = 30$ . The value of assets is largely firm specific (the slope  $\Theta_1 = 1$  is much larger than  $\Theta_1^* = 0.1$ ), but outsiders’ ability to generate cash with the firm’s assets, which determines its liquidation value, is much superior in low states (the intercept  $\Theta_0^* = 4$  is much larger than  $\Theta_0 = -4$ ), hence abandonment is eventually optimal (Assumption 1).

For each type of debt, Table 2 exhibits the ex-ante optimal (a) debt contract (b) capital structure at entry. It then gives the resulting (c) firm value, (d) credit spreads, (e) risk premium, as well as (f) first renegotiation trigger levels and (g) liquidation trigger levels.

Differences in (a) optimal debt contract and (b) optimal capital structure are very substantial. Single creditor debt with opportunistic shareholder behavior being the most threatening type of debt for creditors, the optimal leverage is in that case 29%. This is 63% lower than the optimal leverage with dispersed creditors + optimal guaranteeing which is 80%. Dispersed creditors are vulnerable to dilution threats, but are largely protected from strategic default threats. As a precommitment device against excessive shareholder opportunistic behavior, issuing publicly traded debt enhances the borrowing ability of the firm and because creditors are more willing to lend, yields a much higher ex-ante optimal level of borrowing.

Differences in (c) firm value are surprisingly small by comparison. In spite of the considerable increase in leverage, the optimal type of debt (here dispersed creditors plus optimal

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<sup>26</sup>This is the mean effective tax advantage of debt  $\tau = 1 - (1 - \tau_c)(1 - \tau_V)/(1 - \tau_L)$  found by Graham (1996) for the period 1981-1992.

guaranteeing) only yields a 3 % improvement in total firm value. The timing of events is nevertheless very different. With dispersed creditors plus optimal guaranteeing (*f*) the first renegotiation takes places much later than with a single creditor, but (*g*) the inefficiency of early liquidation is much more severe than in any other case because of the higher leverage.

Importantly, compared with non-renegotiable debt, (*d*) credit spreads and (*e*) risk premia are 2 1/2 to 3 times larger, and this is true both for dispersed creditors and for single-creditor debt, considering optimal contracts in each case. The main criticism of Merton-type debt valuation models (including the Leland model), which consider non-renegotiable debt, is that they fail to generate the level of credit spreads observed in the market. Here, once the capital structure ex-ante optimization is taken into account, our dispersed creditor model generates a credit spread of 298 bps. This is much larger than the 104 bps credit spread obtained with non-renegotiable debt. What is even more significant, it is also larger than the 257 bps credit spread obtained with single-creditor debt. The latter difference is explained predominantly by the higher leverage, but also the higher default risk due to early abandonment. Thus, the correct specification of renegotiable dispersed debt may contribute to an explanation of the credit spread puzzle.

Clearly, these numbers depend on the input parameters for the firm,  $\{x; \Theta_0; \Theta_1; \Theta_0^*; \Theta_1^*\}$ , and the economic environment,  $\{\mu; \sigma; \rho; \tau\}$ . We do not extend our numerical simulations, since our intention is merely to illustrate a qualitative insight. Unrecorded simulations with different input parameters have, however, consistently yielded the same message: Optimal capital structure, default risk premia and credit spreads are substantially different of the pricing model specification correctly accounts for the *multiplicity of creditors* and/or *initial guaranteed* liquidation rights. In other words, if the pricing model wrongly specifies the type of debt, the estimated optimal capital structure could be half, and the estimated credit spread could be a third of the correct one.

## VII. Prioritization and Dilution Threats in Practice

This Section discusses the consistency of our results with empirical evidence about exchange offers and priority of claims.

### A. Priority of Claims and its Role in Debt Exchanges

The most important form of a guaranteed liquidation right in practice, akin to  $G_n^*(x)$  in our model, is certainly *collateral*. A lien gives the creditor exclusive access to the sales proceeds of the collateralized asset, and hence guaranteed priority. The use of secured credit is widespread: In a comprehensive study, Barclay and Smith (1995) report that on average 36 % of all debt claims of listed firms in the period 1981-95 were secured, making secured debt the most important category of debt even for large, stock market-listed firms.

But *senior* debt often also has the status of guaranteed liquidation rights, since typically not all of the liquidation value of a firm can be pledged as collateral. Growth opportunities

and other intangible assets will lead to sales proceeds in liquidation, but the ownership of these assets cannot be separated from the rest of the firm, and hence no lien on them can be enforced. Under Chapter 7 of the US Bankruptcy Code of 1978, the estate value of the non-collateralized assets will be distributed pro rata among all claims which have been allowed. While all claims enjoy in principle equality, §510 of the Bankruptcy Code recognizes explicitly contractual agreements between creditors giving priority to senior creditors at the expense of junior claimants.

Empirical investigations confirm that secured claims and seniority agreements are largely enforced in practice. Weiss (1990) reports in an analysis of 37 bankruptcy cases that priority is almost always maintained with respect to secured creditors. Altman and Kishore (1996) calculate that across all reported corporate bond defaults in the period 1978-95, the mean recovery rate on secured debt is nearly double that of subordinated debt, and that of senior unsecured debt is more than 50 % higher than that on subordinated debt.<sup>27</sup> In a sample of Chapter 11-reorganizations, Franks and Torous (1989) find substantial deviations from absolute priority, benefiting both equity and junior debt classes, but they also note that (p. 755) their “results suggest that unsecured creditors receive only a small fraction of what secured creditors obtain”. Franks and Torous (1994) show that in Chapter 11-proceedings, junior claimants recover on average only 28,9 % of their face value, while senior creditors recover 47 % and secured creditors, 80,1 %.<sup>28</sup>

Our model predicts<sup>29</sup> that when renegotiating dispersed debt claims, firms will try to increase the level of guaranteed priority rights and of collateral. The empirical literature shows indeed that higher priority is routinely attributed in a majority or large proportion of cases.<sup>30</sup> As Gertner and Scharfstein (1991) note, these samples may understate the true extent of prioritization because higher priority, whether explicit or implicit in the covenants, may often not be discernible from the publicized terms of the exchange offer.

In line with our model, James (1996) reports that the larger the *number* of contracts, the larger the amount of *senior* debt offered. Equally consistent with the predictions, James

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<sup>27</sup>Since their calculations do not control for the actual debt structure of defaulting firms, one would expect that these numbers underestimate the difference in recovery rates *within* a single firm that has both secured (or senior) and junior claims outstanding.

<sup>28</sup>When interpreting Chapter 11 data, one should note that 2/3 of securityholders in each class of claims must accept the reorganization plan, making deviations from absolute priority more likely than in liquidation. Also, Chapter 11 filings cannot automatically be equated to an abandonment decision  $y$  in the terminology of our model. Successful Chapter 11-reorganization leaves the firm as a going-concern and attributes a positive value to shareholders, and in many cases its economic significance is rather closer to a debt exchange offer round in our model. This is notably true for prepackaged bankruptcies. Peterson (1993) reports that 12 % of exchange and tender offers in the 1990-92 period were filed as prepackaged bankruptcies.

<sup>29</sup>Other papers including Gertner and Scharfstein (1991) also predict this.

<sup>30</sup>The following fractions where more senior debt is offered are reported: James (1996) 64 %, Chatterjee, Dhillon and Ramirez (1995) 76 %, Brown, James and Mooradian (1993) 43 %, Asquith, Gertner and Scharfstein (1994) 41 %.

(1996) and Gilson (1997) find that the amount of concessions *decreases* in the number of debt contract outstanding.

### *B. Collateralization in Debt Restructurings*

There is also evidence that the level of *collateral* is raised as a consequence of debt restructuring efforts. Brady bonds are a prominent example in this respect. Arguably, holdout problems are even more severe in sovereign debt restructurings than in corporate workouts.<sup>31</sup> After several attempts to reschedule sovereign debt in the late 80s had failed, the solution that finally succeeded involved a substantial collateral portion. Under the Brady plan of 1989, the par value and a rolling window of three coupon payments are guaranteed by collateral deposited at the Federal Reserve of New York.

In the corporate area, there is evidence that creditors demand more secured debt as firms approach the financial distress bound. For firms *outside* distress, levels of secured debt are significant, but far from exhausting all collateralizable assets. Houston and James (1996) find that virtually all publicly listed US-firms relied on bank credit to some extent, but only a third had secured bank debt. By contrast, James (1995, 1996) points out that virtually all bank debt of financially *distressed* firms is secured.

Our model predicts that highly collateralized firms with dispersed debt will have little success in renegotiating the claims out-of-court. Consistent with this, Asquith, Gertner and Scharfstein (1994) find that the chances of a distressed junk bond issuer to avoid bankruptcy via debt renegotiation decreases significantly in the fraction of debt which is secured. Their regressions indicate that “a shift from none of the debt being secured to all of it being secured increases the estimated probability of bankruptcy by 0.34”.

Moreover, we predict that debtors would use all instruments to dilute the liquidation rights of creditors, before they would finally abandon operations. Note that our model does not predict that only secured creditors will recover value in liquidation; rather, it predicts that the fraction of liquidation proceeds recouped by non-secured creditors increases in the value of non-collateralizable assets.<sup>32</sup> Consistent with this view, Barclay and Smith (1995) find that across all listed US-firms the fraction of secured debt significantly decreases in the market-to-book ratio, the proxy for growth opportunities non-collateralizable assets.

### *C. Other Dilution Devices*

While we focus on debt-for-debt exchanges in this paper, we wish to emphasize that other vehicles are frequently deployed, generating much the same economic coercion effect as priority, and which therefore should be regarded as equivalent dilution threats.

Many debt-for-debt exchange offers *shorten the maturity* of the debt claims, thereby

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<sup>31</sup>In sovereign workouts, among other things, large debtholders facilitating renegotiation are usually absent.

<sup>32</sup>In our model, non-secured creditors should receive normally a positive payoff in liquidation, in all but those cases where all liquidation proceeds could be fully collateralized.

increasing the expected value of a tendering creditor's residual claim at the expense of holdouts. Gertner and Scharfstein (1991) report that 75 % of debt-for-debt exchange offers in their sample propose maturity shortening. The extreme form of maturity shortening is a *cash payment*. Tender offers, where only cash is offered, are very common: Chatterjee, Dhillon and Ramirez (1995) and Peterson (1993) report as many tender offers as exchange offers in their samples of distressed workouts; and in Kahan and Tuckman (1993), the number of tender offers is three times higher than that of exchange offers. And even when new securities are offered, the exchange offers often propose packages containing new claims as well as a cash component. Gertner and Scharfstein (1991) report that cash was offered in 23 % of all exchange offers. Empirical studies show that consent solicitations are even more popular in tender offers than in exchange offers (Peterson (1993) and Chatterjee, Dhillon and Ramirez (1995)).

A very common dilution threat consists of *asset sales* occurring in parallel to the debt restructuring. The cash proceeds of asset sales can be used to finance a cash tender offer, thus diminishing the value of existing liquidation rights twofold, via the drain on cash reserves and the reduction in the expected liquidation value as assets have been sold off. Gilson (1997) finds that 51 % of firms sold assets during a year in which they renegotiated their debt, and including the preceding year, 69 % had asset sales. Crucially, he finds that these asset sales significantly increase the chances of a firm to get concessions on the debt outstanding. Creating a similar dilution effect, valuable assets are sometimes *spun off* into a different legal entity beyond the reach of existing debtholders.<sup>33</sup>

## VIII. Possible Extensions

### A. Releveraging

In our model, the optimal debt level can be shown to be a monotonically increasing function of  $x_0$ . If  $x_t$  falls, capturing a deterioration in the conditions of the firm, the debt level will be adjusted via repeated renegotiation rounds, as discussed in Section III. In the model with dispersed creditors, it is easy to see why a *debt buyback* is not attractive for the manager: debt would have to be bought back at the full value that a hold-out strategy could secure, i.e. the coupon level until the equilibrium stopping point  $y_s$ . The total payout to debtholders would then be the same in our model as if the debt was fully serviced until  $y_s$ . As a consequence, debt buybacks will never be used, even if possible, because they would only delay the stopping decision beyond the point that is optimal after entry.

By contrast, if  $x_t$  rises beyond the entry level  $x_0$ , the manager wishes to issue additional debt. The firm will want to relever as its earnings reach new historic highs  $x_t$  to benefit from a larger tax shield, as in Goldstein, Ju and Leland (2001).<sup>34</sup> In other words, new growth

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<sup>33</sup>See Amihud, Garbade and Kahan (1999) for an example.

<sup>34</sup>We are grateful to the referee for raising the issue of subsequent debt issues.

opportunities, loosely represented by an increased market value of the firm, should translate into new debt issues.

Debt renegotiation, however, makes the analysis considerably more complicated than Goldstein, Ju and Leland (2001), who do not consider this issue. We are then considering a two-sided problem with debt renegotiation on the downside and releveraging on the upside. Importantly, the firm will not wish to increase leverage at  $x_t > x_0$  exactly to the level that it would have chosen at entry if  $x_t$  had been the entry point. The reason is that the function  $G^*(x)$  chosen initially commits the firm to a more patient approach to debt renegotiation than appears optimal when looked at from  $x_t$ .<sup>35</sup> Even more, if the firm has begun to renegotiate, it would optimally releverage if it recovers, but less aggressively so, as parts of its bargaining chip  $V^*(y_s) - NG^*(y_s)$  has already irreversibly been pledged away in past rounds of renegotiation. Thus, the optimal releveraging policy is history dependent in a very complex fashion.

What would be the optimal seniority level of the new debt? In our model, only two seniority levels exist: claims secured by guaranteed liquidated rights, and unsecured claims. No further distinction has any economic significance, since the firm honors all coupon obligation alike before defaulting, and only guaranteed liquidation rights have value in the case of default. Therefore, and considering that the firm has already more liquidation rights pledged away than it wished to have at the time of the new issue, the new debt would be *entirely unsecured*, and hence either be subordinated to the existing claims or have equal priority. This outcome of our model is reminiscent of the observation that many firms tend to issue junior claims when new growth opportunities arise.

### *B. Bank Debt, Large Debtholders and Mixed Debt Structures*

Firms often have more complicated debt structures than in our model where there is initially just a single class of public debt. Many firms have mixed debt structures combining concentrated blocks of debt, like bank debt and private placements, and dispersed or publicly traded debt. The interaction between bank debt restructurings and the terms and success probability of public debt exchange offers is important in practice, as has been observed notably by Asquith, Gertner and Scharfstein (1994) and James (1996).<sup>36</sup>

Our model would predict that a bank would agree to a debt-for-equity swap if it is the single important creditor of the firm, but it will be reluctant to do so if there are competing dispersed debt claims for fear of “buoying up” the value of these claims. This seems to be consistent with the evidence.<sup>37</sup>

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<sup>35</sup>The abandonment point  $y_s$  that maximizes the initial static trade-off is normally increasing in  $x_0$ .

<sup>36</sup>Asquith, Gertner and Scharfstein (1994) report for their sample of distressed junk bond issuers that 86 % restructure bank debt, but only 50 % attempt to reschedule their public debt. A fifth of the bank debt restructurings in their sample lead to an increase in bank collateral.

<sup>37</sup>Brown, James and Mooradian (1993) report that firms have public debt outstanding in 75 % of the cases where banks increase the collateral level, but only in 27 % of the cases where bank accept equity positions.



Including banks' renegotiation strategies provides, in our view, the most promising avenue to incorporate debt-for-equity swaps into our model. The frequency of debt-for-equity swaps presents a puzzle for a theoretical model like ours based on the hold-out incentives of dispersed debtholders, as Gertner and Scharfstein (1991) note. To account for bank behavior could be an interesting avenue to explain this puzzle, since debt-for-equity offers in bond workouts often are imposed by banks. James (1996) finds that the larger the bank concession, the less senior debt will be offered to bondholders, presumably since exclusive seniority is a condition for banks to make concessions.<sup>38</sup>

### C. Asset Characteristics and Alternative Commitment Devices

Our analysis has assumed that the manager can choose the function  $G_0^*(x)$ , within the feasibility bounds, so as to commit to abandonment at the ex ante preferred point,  $y_s$ . We wish to emphasize that the nature of collateralizable assets certainly puts limitations on the ability to define this function. Collateral consists of a collection of assets whose liquidation values is determined by their *physical* characteristics. In practice, we would certainly expect that physical asset characteristics impose substantial restrictions on the shareholder's ex ante choice of the initial guaranteed liquidation rights,  $G_0^*(x)$ . The assets with liquidation value  $V_0^*(x)$  may not be malleable enough to approximate the optimal function  $G_0^*(x)$ .

There exist, however, a couple of interesting *contractual* devices having the potential to solve the manager's problem, namely to commit her to not default before the second best point  $y_s$ . We want to briefly discuss two rather well-known contract features, callability and maturity, that have the potential to make the manager more patient and to increase ex ante firm value.

(i) *Callable Debt*. Suppose the firm issues fully collateralized debt at  $x_0$ , as a combination of several bonds. Some bond issue have zero initial collateral, but there is also at least one bond which carries a high aggregate collateral and which is callable: The shareholder can call the zero bond at any time for a fixed call price. This call feature is attached only to the bonds which carry collateral. Therefore, only after calling a collateralized bond can the shareholder use the portion of the liquidation value over which she just gained discretion in order to renegotiate coupon concession from the holders of the non-collateralized coupon bonds. Consequently, setting the right call prices, the shareholder can credibly commit to

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James (1996) finds that concomitant bank concessions significantly increase the concessions obtained in exchange offers of dispersed debt, and the odds for exchange offers to succeed. The evidence in non-US financial systems is consistent with the view that banks increase their collateral as firms go into financial distress, as Harhoff and Koerting (1998) report for Germany and Franks and Sussman (2000) for the UK.

<sup>38</sup>For example, banks might impose debt-for-equity swaps on public creditors, as a precondition for their making of concessions, as a means to avoid institutional barriers on their own equity holdings. A debt-for-equity swap can be engineered in such a way that it is based on a dilution threat, for example if high cash flow promises are made, which implicitly lower the liquidation value of holdouts. A debt-for-equity swap might also be attractive to signal the poor prospects of the distressed firm, as explained by Brown, James and Mooradian (1993).

exercise the calls exactly at the right trigger points.

*(ii) Maturity Structure.* A short debt maturity represents a vehicle committing the debtor to abstain from a premature use of the option to renegotiate. The intuition is as follows: Under short maturity, a given fraction of the debt will have to be refinanced between the exchange offer trigger point and the final abandonment trigger level. The earlier the exchange offer trigger point, the higher this probability. Therefore, investors anticipate that the probability for enjoying full repayment of the principal is increasing if the renegotiation proposal is launched early rather than late. As a result, debtholders will require a more generous exchange offer in order to be willing to surrender their contracts.

The choice of average maturity determines what fraction of debt is expected to be refinanced between renegotiation and abandonment: If maturity is very long, none of the debt is expected to be refinanced, with the incentive consequences studied earlier. If maturity is very short, almost all of the debt is fully and instantaneously refinanced, so the debtor has no incentive at all to propose exchange offers, just as with fully collateralized debt.

## IX. Conclusion

This paper presents a fully dynamic analysis of debt renegotiation with many creditors. In this framework, it becomes clear that demands for concessions lack intertemporal consistency if they are not backed up by additional liquidation rights which are safe from continued expropriation. This insight provides a natural explanation for the notion that creditors derive some protection against opportunistic default threats if debt is widely dispersed.

The repeated and dynamic nature of our analysis enables us to identify the debtor's commitment problem: to make debt renegotiation possible, the debtor must be able to commit to liquidation rights which are guaranteed. This provides a justification for the existence of *(i)* secured liquidation rights (collateral as well as debt-seniority, depending on the intangibility of firm assets), *(ii)* protective covenants limiting debt-issuance and *(iii)* seemingly coercive voting practices such as exit-consent solicitations, which can all be valuable by fostering renegotiability.

The possibility of subsequent debt renegotiation rounds severely limits the size of concessions that an opportunistic debtor can obtain. This can be attractive from an ex ante perspective: Debt dispersion protects creditors from excessive opportunistic expropriation. As a result, the debt capacity is larger than in the single-creditor case. This paper identifies the capacity to sustain a higher level of borrowing and thus a higher debt tax shield as an important motive behind the issue of dispersed (public) debt.

We find that treating dispersed debt as if it was concentrated debt leads to substantial pricing errors. Anything else would have been sheer coincidence, given that debt renegotiation relies on such fundamentally different economic mechanisms. We furthermore find large

differences in both the optimal capital structure policy and the debt riskiness, although the resulting ex ante firm value is about equal.

An important insight of our model is that it cannot only explain substantial default spreads, but that the default spread may even be larger than in the case of a single creditor. This difference is explained both by the higher level of debt and by the fact that the optimal abandonment decision is premature, which raises the risk for existing debt claims.

Understanding the dynamics of mixed debt structures, combining concentrated and dispersed debt claims as well as multiple layers of priority, presents an empirically pertinent and theoretically intriguing agenda for future research.

## Appendix

**Proof of Lemma 1:** Assume that  $n_1$  debtholders have accepted one offer, even though the shareholder can make a second offer in the future. These senior creditors are receiving a coupon  $\delta_1$  since accepting the first offer, at  $\underline{x}_1$ . Now, the most opportunistic second offer the shareholder could make is such that tendering is marginally better than holding out,

$$D_{i \in \mathcal{T}_2}^{(2)}(\underline{x}_2) = D_{i \in \mathcal{H}_2}^{(2)}(\underline{x}_2).$$

Therefore, just before this second offer, the value of these debtholders' claim can be reduced to

$$D_{i \in \mathcal{T}_1}^{(1)}(\underline{x}_2^+) = \frac{\delta_1}{\rho} + \left[ D_0^*(y) - \frac{\delta_1}{\rho} \right] \mathcal{P}(\underline{x}_2 \triangleright y).$$

The value of their claim in this current regime 1, i.e. for  $\tilde{x}_t \in [\underline{x}_2, \underline{x}_1)$ , is therefore

$$D_{i \in \mathcal{T}_1}^{(1)}(x_t) = \frac{\delta_1}{\rho} + \left[ D_0^*(y) - \frac{\delta_1}{\rho} \right] \mathcal{P}(x_t \triangleright y).$$

However, bondholders who did not tender during the first offer currently hold a claim worth

$$D_{i \in \mathcal{H}_1}^{(1)}(x_t) = \frac{\delta_0}{\rho} + \left[ D_0^*(y) - \frac{\delta_0}{\rho} \right] \mathcal{P}(x_t \triangleright y).$$

Therefore, when the first offer was made, the tendering condition was clearly violated

$$D_{i \in \mathcal{T}_1}^{(1)}(\underline{x}_1) < D_{i \in \mathcal{H}_1}^{(2)}(\underline{x}_1).$$

This contradicts the initial assumption, that  $n_1$  bondholders have accepted one offer. **QED.**

**Proof of Lemma 2:** Consider that the  $k^{\text{th}}$  tendering condition is binding:

$$D_{i \in \mathcal{T}_k}^{(k)}(\underline{x}_k) = D_{i \in \mathcal{H}_k}^{(k)}(\underline{x}_k) = \frac{\delta_{k-1}}{\rho} + \left[ G_{k-1}^*(y_s) - \frac{\delta_{k-1}}{\rho} \right] \mathcal{P}(\underline{x}_k \triangleright y_s).$$

In regime  $k$ , i.e. for  $\tilde{x}_t \in (\underline{x}_{k+1}, \underline{x}_k]$ , debtholders who have tendered receive a coupon  $\delta_k$  until the  $k+1^{\text{th}}$  offer, which occurs at  $\underline{x}_{k+1}$ . The value of their claim is

$$D_{i \in \mathcal{T}_k}^{(k)}(x_t) = \frac{\delta_k}{\rho} + \left[ D_{i \in \mathcal{H}_k}^{(k)}(\underline{x}_{k+1}^+) - \frac{\delta_k}{\rho} \right] \mathcal{P}(x_t \triangleright \underline{x}_{k+1}).$$

We have two expressions for the value of this claim, just after the  $k^{\text{th}}$  offer, i.e. for  $(x_t) = (\underline{x}_k^+)$ ,

$$\frac{\delta_k}{\rho} + \left[ D_{i \in \mathcal{T}_k}^{(k)}(\underline{x}_{k+1}^+) - \frac{\delta_k}{\rho} \right] \mathcal{P}(\underline{x}_k \triangleright \underline{x}_{k+1}) = \frac{\delta_{k-1}}{\rho} + \left[ G_{k-1}^*(y_s) - \frac{\delta_{k-1}}{\rho} \right] \mathcal{P}(\underline{x}_k \triangleright y_s).$$

Therefore the value of tendering debtholders' claim just before the  $k+1^{\text{th}}$  offer is already determined,

$$D_{i \in \mathcal{T}_k}^{(k)}(\underline{x}_{k+1}^+) = \frac{\delta_k}{\rho} + \left( \frac{\delta_{k-1} - \delta_k}{\rho} + \left[ G_{k-1}^*(y_s) - \frac{\delta_{k-1}}{\rho} \right] \mathcal{P}(\underline{x}_k \triangleright y_s) \right) \frac{1}{\mathcal{P}(\underline{x}_k \triangleright \underline{x}_{k+1})}.$$

Now, when the  $k + 1^{th}$  exchange offer is triggered at  $(x_t) = (\underline{x}_{k+1})$ , the most opportunistic offer the shareholder could make is such that the  $k + 1^{th}$  tendering constraint is binding

$$D_{i \in \mathcal{T}_{k+1}}^{(k+1)}(\underline{x}_{k+1}) = D_{i \in \mathcal{H}_{k+1}}^{(k+1)}(\underline{x}_{k+1}) = \frac{\delta_k}{\rho} + \left[ G_k^*(y_s) - \frac{\delta_k}{\rho} \right] \mathcal{P}(\underline{x}_{k+1} \triangleright y_s).$$

To be time consistent, the  $k^{th}$  exchange offer must guarantee that the future value of tendering debtholders' claim just before the  $k + 1^{th}$  offer is greater or equal to its value if the shareholder decides then to make the most opportunistic  $k + 1^{th}$  offer possible. The  $k^{th}$  exchange offer must ensure that

$$\begin{aligned} \frac{\delta_k}{\rho} + \left( \frac{\delta_{k-1} - \delta_k}{\rho} + \left[ G_{k-1}^*(y_s) - \frac{\delta_{k-1}}{\rho} \right] \mathcal{P}(\underline{x}_k \triangleright y_s) \right) \frac{1}{\mathcal{P}(\underline{x}_k \triangleright \underline{x}_{k+1})} \\ \geq \frac{\delta_k}{\rho} + \left[ G_k^*(y_s) - \frac{\delta_k}{\rho} \right] \mathcal{P}(\underline{x}_{k+1} \triangleright y_s). \end{aligned}$$

The  $k^{th}$  exchange offer must involve an increase in guaranteed liquidation rights at least equal to

$$G_k^*(y_s) - G_{k-1}^*(y_s) \geq \left[ \frac{\delta_{k-1} - \delta_k}{\rho} \right] \frac{[1 - \mathcal{P}(\underline{x}_k \triangleright y_s)]}{\mathcal{P}(\underline{x}_k \triangleright y_s)}. \quad \text{QED.}$$

**Proof of Lemma 3:** The proof is by backwards induction. First, we show that the tendering constraint must be binding in the last (the  $K^{th}$ ) exchange offer. Then, we show that by induction, the tendering constraint must also be binding in all previous exchange offers. After the  $K^{th}$  offer, the value of a creditor who successfully tenders in this last round is given by:

$$D_{i \in \mathcal{T}_K}^{(K)}(x_t) = \frac{\delta_K}{\rho} + \left[ \frac{V^*(y) - \sum_{j=1}^K (n_{j-1} - n_j) G_{j-1}^*(y)}{n_K} - \frac{\delta_K}{\rho} \right] \mathcal{P}(x_t \triangleright y) \quad (51)$$

Expression (51) takes into account that, after the last offer, any liquidation rights which are not guaranteed to creditors previously held out, are equally distributed among the  $n_K$  creditors who successfully tender in the last round, hence  $D_K^*(y) = V^*(y) - \sum_{j=1}^K (n_{j-1} - n_j) G_{j-1}^*(y)$ . The value in the last regime of a creditor held out in the  $K^{th}$  offer is as stated in (7).

To simplify notation, we consider strategies where  $n_k = K$ , for all  $k \in \{1; \dots; K\}$ , i.e. the debtor offers  $K$  new contracts in each round. This is without loss of generality.

The optimal abandonment trigger level,  $y_s$ , solves

$$y_s = \arg \max_y \left\{ (1 - \tau) \left[ -\Pi(y) + N \frac{\delta_K}{\rho} \right] \mathcal{P}(x \triangleright y) \right\}.$$

The value of all debt claims *right after* and *right before* the  $K^{th}$  offer are equal:

$$N \frac{\delta_K}{\rho} + \left[ V^*(y_s) - N \frac{\delta_K}{\rho} \right] \mathcal{P}(\underline{x}_K \triangleright y_s) = \sum_{i=1}^N D_{i \in \mathcal{T}_{K-1} \cup \mathcal{H}_{j \leq K-1}}^{(K-1)}(\underline{x}_K^+).$$

Therefore, the value of all debt claims before this  $K^{th}$  offer, i.e. in regime  $K - 1$  for  $\tilde{x}_t \in [\underline{x}_K, \underline{x}_{K-1})$ ,

$$\begin{aligned} \sum_{i=1}^N D_{i \in \mathcal{T}_{K-1} \cup \mathcal{H}_{j \leq K-1}}^{(K-1)}(x_t) &= N \frac{\delta_{K-1}}{\rho} + \left[ \sum_{i=1}^N D_{i \in \mathcal{T}_{K-1} \cup \mathcal{H}_{j \leq K-1}}^{(K-1)}(\underline{x}_K^+) - N \frac{\delta_{K-1}}{\rho} \right] \mathcal{P}(x_t \triangleright \underline{x}_K), \\ &= N \frac{\delta_{K-1}}{\rho} + \left[ N \frac{\delta_K}{\rho} + \left( V^*(y_s) - N \frac{\delta_K}{\rho} \right) \mathcal{P}(\underline{x}_K \triangleright y_s) - N \frac{\delta_{K-1}}{\rho} \right] \mathcal{P}(x_t \triangleright \underline{x}_K). \end{aligned}$$

Next, we develop an argument which holds for *any given* sequence of guaranteed liquidation rights changes,  $G_k^*(x)$ , for  $k = 0, 1, \dots, K$ . This argument must then also be valid for any *optimal* sequence of guaranteed liquidation rights values. Replacing in the shareholder's problem in regime  $K - 1$ , for  $\tilde{x}_t \in [\underline{x}_K, \underline{x}_{K-1}]$ :

$$\begin{aligned} \max_{(\delta_K, \underline{x}_K)} \quad & (1 - \tau) \Pi(x_t) - (1 - \tau) \Pi(y_s) \mathcal{P}(x_t \triangleright y_s) - N(1 - \tau) \frac{\delta_{K-1}}{\rho} [1 - \mathcal{P}(x_t \triangleright \underline{x}_K)] \\ & - N(1 - \tau) \frac{\delta_K}{\rho} [\mathcal{P}(x_t \triangleright \underline{x}_K) - \mathcal{P}(x_t \triangleright y_s)] \end{aligned} \quad (52)$$

$$\text{s.t.} \quad G_K^*(y_s) - G_{K-1}^*(y_s) \geq \frac{(\delta_{K-1} - \delta_K)}{\rho} \frac{[1 - \mathcal{P}(\underline{x}_K \triangleright y_s)]}{\mathcal{P}(\underline{x}_K \triangleright y_s)}, \quad (53)$$

$$x_t \geq \underline{x}_K \geq y_s \quad (54)$$

$$y_s = \arg \max \left\{ \left[ V^*(y) - N G_K^*(y) - (1 - \tau) \Pi(y) + N(1 - \tau) \frac{\delta_K}{\rho} \right] \mathcal{P}(x \triangleright y) \right\}, \quad (55)$$

We know that the second part of the constraint (54) cannot be binding, because in regime  $K - 1$ ,  $\delta_{K-1} < \delta_K$ . We can disregard this constraint when setting up the (Kuhn-Tucker) Lagrangian of this problem, which we write as:

$$\begin{aligned} \max_{\delta_K, \underline{x}_K} L = \quad & (1 - \tau) \Pi(x_t) - (1 - \tau) \Pi(y_s) \mathcal{P}(x_t \triangleright y_s) - N(1 - \tau) \frac{\delta_{K-1}}{\rho} [1 - \mathcal{P}(x_t \triangleright \underline{x}_K)] \\ & - N(1 - \tau) \frac{\delta_K}{\rho} [\mathcal{P}(x_t \triangleright \underline{x}_K) - \mathcal{P}(x_t \triangleright y_s)] \\ & - \mu \left[ \frac{(\delta_{K-1} - \delta_K)}{\rho} \frac{[1 - \mathcal{P}(\underline{x}_K \triangleright y_s)]}{\mathcal{P}(\underline{x}_K \triangleright y_s)} - (G_K^*(y_s) - G_{K-1}^*(y_s)) \right] - \nu [\underline{x}_K - x_t] \end{aligned}$$

The proof is by contradiction. Suppose to the contrary that the  $K^{\text{th}}$  tendering constraint (53) is *not* binding, i.e.  $\mu = 0$ . We distinguish two cases:

*Case A:*  $\nu = 0$ , i.e. the first part of (54) is *not* binding. Then  $\mu = \nu = 0$ . The (Kuhn-Tucker) Lagrangian becomes:

$$\begin{aligned} \max_{\delta_K, \underline{x}_K} L = \quad & (1 - \tau) \Pi(x_t) - (1 - \tau) \Pi(y_s) \mathcal{P}(x_t \triangleright y_s) - N(1 - \tau) \frac{\delta_{K-1}}{\rho} [1 - \mathcal{P}(x_t \triangleright \underline{x}_K)] \\ & - N(1 - \tau) \frac{\delta_K}{\rho} [\mathcal{P}(x_t \triangleright \underline{x}_K) - \mathcal{P}(x_t \triangleright y_s)] . \end{aligned}$$

Maximizing gives the FOC:

$$\frac{\partial L}{\partial \underline{x}_K} = -N(1 - \tau) \left[ \frac{\delta_{K-1} - \delta_K}{\rho} \right] \frac{\partial \mathcal{P}(\underline{x}_K \triangleright y_s)}{\partial \underline{x}_K}. \quad (56)$$

By construction,  $\delta_K < \delta_{K-1}$ . Hence  $\partial L / \partial \underline{x}_K > 0$ , contradicting the assumption that  $\mu = \nu = 0$ .

*Case B:*  $\nu \neq 0$ , i.e. the first part of (54) is binding. We have then  $\mu = 0$  and  $\nu \neq 0$  and hence  $\underline{x}_K = x_t$ . The (Kuhn-Tucker) Lagrangian becomes:

$$\begin{aligned} \max_{\delta_K, \underline{x}_K} L = \quad & (1 - \tau) \Pi(x_t) - (1 - \tau) \Pi(y_s) \mathcal{P}(x_t \triangleright y_s) \\ & - N(1 - \tau) \frac{\delta_K}{\rho} [1 - \mathcal{P}(x_t \triangleright y_s)] - \nu [\underline{x}_K - x_t]. \end{aligned}$$

Maximizing gives the FOC:

$$\frac{\partial L}{\partial \delta_K} = -N \frac{(1-\tau)}{\rho} [1 - \mathcal{P}(x_t \triangleright y_s)] + \frac{\partial}{\partial y_s} \left\{ (1-\tau) \left[ -\Pi(y_s) + N \frac{\delta_K}{\rho} \right] \mathcal{P}(x \triangleright y_s) \right\} \frac{\partial y_s}{\partial \delta_K} \quad (57)$$

which simplifies to  $\partial L / \partial \delta_K = -N(1-\tau)/\rho [1 - \mathcal{P}(x_t \triangleright y_s)]$  by the envelope theorem. Hence  $\partial L / \partial \delta_K < 0$ , contradicting the assumption that  $\mu = 0$ .

Thus, in Case A and in Case B there is a contradiction to the assumption that  $\mu = 0$ . This concludes the proof that the  $K^{\text{th}}$  tendering constraint is binding.

It remains to develop the induction argument. Consider regime  $k < K$ , with a coupon of  $\delta_k$ . The proof by contradiction mirrors the one for the  $K^{\text{th}}$  regime. Consider Case A where  $\mu = \nu = 0$ . At any state  $x_t$  in the regime  $k$ , the first order condition of the Lagrangian yields as in Eq. (56):

$$\frac{\partial L}{\partial \underline{x}_{k+1}} = -N(1-\tau) \left[ \frac{\delta_k - \delta_{k+1}}{\rho} \right] \frac{\partial \mathcal{P}(x_t \triangleright \underline{x}_{k+1})}{\partial \underline{x}_{k+1}},$$

giving a contradiction as  $\frac{\partial L}{\partial \underline{x}_{k+1}} \neq 0$  is implied by  $\delta_k > \delta_{k+1}$ . Hence if  $\mu = 0$ , necessarily  $\nu \neq 0$ . Consider Case B where  $\mu = 0$  but  $\nu \neq 0$ . At any state  $x_t$  in the regime  $k$ , the first order condition of the Lagrangian becomes in analogy to condition (57):

$$\frac{\partial L}{\partial \delta_k} = -N \frac{(1-\tau)}{\rho} [1 - \mathcal{P}(x_t \triangleright y_s)] + \frac{\partial}{\partial y_s} \left\{ (1-\tau) \left[ -\Pi(y_s) + N \frac{\delta_k}{\rho} \right] \mathcal{P}(x \triangleright y_s) \right\} \frac{\partial y_s}{\partial \delta_k},$$

which simplifies to  $\partial L / \partial \delta_k = -N(1-\tau)/\rho [1 - \mathcal{P}(x_t \triangleright y_s)] < 0$  by the envelope theorem, contradicting  $\mu = 0$ . **QED.**

**Result 1 (Invariance of ex post optimal abandonment strategy)** *There exists a feasible strategy where the shareholder will abandon exactly at  $y_s$ , where  $y_s$  denotes the shareholders' value maximizing abandonment trigger point right after entry.*

**Proof:** Since all exchange offers along an optimal exchange offer strategy  $\mathbf{s}$  will be binding by virtue of Lemma 3,  $S^{(0)}(x_t)$  can be written as in (23). Then consider the following single exchange offer strategy  $\mathbf{s}$  where  $K = 1$ . Let  $\hat{\delta}$  be a coupon such that

$$y_s = \arg \max_y \left\{ V(x_t | y) - N \left[ \frac{\hat{\delta}}{\rho} + \left( \frac{V^*(y)}{N} - \frac{\hat{\delta}}{\rho} \right) \mathcal{P}(x_t \triangleright y) \right] \right\}. \quad (58)$$

That is, if  $\hat{\delta}$  is the average coupon after debt's liquidation rights are fully guaranteed (non-renegotiable), then the shareholder will ultimately abandon exactly at the point  $y_s$ . Next, define  $\hat{x}$  as the point such that

$$V^*(y_s)/N - G_0^*(y_s) = \frac{(\delta_0 - \hat{\delta})}{\rho} \frac{[1 - \mathcal{P}(\hat{x} \triangleright y_s)]}{\mathcal{P}(\hat{x} \triangleright y_s)}. \quad (59)$$

That is, if the shareholder proposes the first exchange offer exactly at  $\hat{x}$  and if the shareholder proposes to guarantee all not initially guaranteed liquidation rights,  $V^*(x) - N G_0^*(x)$ , in this offer, he obtains an coupon reduction of  $N(\delta_0 - \hat{\delta})$  in exchange.

Consider then a single exchange offer strategy  $(\hat{x}, \hat{\delta}, n_1, \mathcal{D}_1)$ , with a new contract of  $\mathcal{D}_1 = (\hat{\delta}_1, V^*(x) - NG_0^*(x)/n_1)$ , where  $\hat{\delta}_1 = \delta_0 - N(\delta_0 - \hat{\delta})/n_1$ . This implies that the average coupon after the exchange is  $\hat{\delta} = n_1\hat{\delta}_1 + (N - n_1)\delta_0$ . First, by construction of  $\hat{\delta}$  in Eq. (58), this coupon reduction is just enough to ensure that the shareholder's ex post optimal abandonment point will be  $y_s$ . Therefore, this single exchange offer strategy is optimal at any point *prior* to reaching  $\hat{x}$  for the first time, since it implies that the upper bound of the equity value is reached,  $\bar{S}(x_t)$ . Second, this single exchange offer strategy is optimal at any point *after* reaching  $\hat{x}$  for the first time, since then debt's liquidation rights are fully guaranteed and no further exchange offers can succeed.

The existence proof is finished by noting that  $\mathcal{P}(x \triangleright y_s)$  is strictly decreasing and continuous in  $x$ , and  $\mathcal{P}(x \triangleright y_s) \rightarrow 1$  as  $x \rightarrow \hat{y}$ . For any coupon  $\hat{\delta} \leq \delta_0$  and debt contract  $(\delta_0, G_0^*(x))$  such that

$$V^*(y_s)/N - G_0^*(y_s) < \frac{(\delta_0 - \hat{\delta})}{\rho} \frac{[1 - \mathcal{P}(x_0 \triangleright y_s)]}{\mathcal{P}(x_0 \triangleright y_s)}, \quad (60)$$

there will be an interior solution  $\hat{x} \in (y_s, x_0)$  at which the shareholder can launch an optimal single exchange offer.

Finally, note that other optimal exchange offer strategies may exist as well. Any optimal strategy  $s$  must, in any regime  $k$ , satisfy the condition:

$$\hat{y} \equiv \arg \max_y \left\{ V(x_t | y) - N \left[ \frac{\hat{\delta}_k}{\rho} + \left( \hat{G}_k^*(y) - \frac{\hat{\delta}_k}{\rho} \right) \mathcal{P}(x_t \triangleright y) \right] \right\}. \quad (61)$$

$N \hat{\delta}_k = \sum_{j=1}^k (n_{j-1} - n_j) \delta_{j-1} + n_k \delta_k$  is the aggregate coupon and  $N \hat{G}_k^*(x) = \sum_{j=1}^k (n_{j-1} - n_j) G_{j-1}^*(x) + n_k G_k^*(x)$  is the aggregate value of guaranteed liquidation rights in the  $k^{th}$  regime. **QED.**

**Proof of Equation (29):** Consider the following deviation strategy for the manager: do not renegotiate, and abandon at  $y_f(\delta_0)$ . This gives the following incentive constraint

$$S^{(0)}(x_0 | y_s) \geq S_f^{(0)}(x_0 | y_f). \quad (62)$$

Clearly,  $G_0^*(x)$  can always be designed in a way such that any other deviation strategy of the manager is less attractive than abandoning either at  $y_f$  (without renegotiating) or at  $y_s$  (after optimal renegotiation). To see this, consider a differentiable approximation of the following function  $G_0^*(x)$ :  $G_0^*(x) = NV^*(x)$  for all  $x > y_s$  and  $G_0^*(y_s) = 0$ .

Therefore, the lowest ex ante equity value that is incentive-compatible is the value where (62) is just binding. **QED.**

**Proof of Equation (30):** Condition (29) can be written as

$$NG_0^*(y_s) = V^*(y_s) - \Pi(y_s) + N \frac{\delta_0}{\rho} + \left( \Pi(y_f) - N \frac{\delta_0}{\rho} \right) \frac{1}{\mathcal{P}(y_f \triangleright y_s)}. \quad (63)$$

Using (63) to rewrite  $D^{(0)}(x_0)$  gives

$$D^{(0)}(x_0) = \frac{\delta_0}{\rho} + \left[ \frac{\Pi(y_f)}{N} - \frac{\delta_0}{\rho} \right] \mathcal{P}(x_0 \triangleright y_f) + \frac{1}{N} [V^*(y_s) - \Pi(y_s)] \mathcal{P}(x_0 \triangleright y_s). \quad (64)$$



Differentiating (64) we obtain

$$\begin{aligned} \frac{dD^{(0)}(x_0)}{d\delta_0} &= \frac{1}{\rho} + \left( \frac{\partial \Pi(y_f)}{\partial y_f} \frac{\partial y_f}{\partial \delta_0} - \frac{N}{\rho} \right) \frac{\mathcal{P}(x_0 \triangleright y_f)}{N} + \left( \frac{\Pi(y_f)}{N} - \frac{\delta}{\rho} \right) \frac{\partial \mathcal{P}(x_0 \triangleright y_f)}{\partial \delta_0} \\ &= \frac{1}{\rho} (1 - \mathcal{P}(x_0 \triangleright y_f)) > 0. \quad \mathbf{QED.} \end{aligned} \quad (65)$$

**Proof of Lemma 4:** Note that

$$\frac{\partial S(x_0 | y_s)}{\partial y_s} = \frac{\partial (U(x_0 | y_s) - (1 - \tau)ND^{(0)}(x_0))}{\partial y_s} = 0, \quad (66)$$

hence

$$\frac{\partial D^{(0)}(x_0)}{\partial y_s} = \frac{1}{N(1 - \tau)} \frac{\partial U(x_0 | y_s)}{\partial y_s}. \quad (67)$$

Note also that  $\partial U(x_0 | y_s)/\partial \delta_0 = 0$ . Using these equalities, the derivative

$$\begin{aligned} \frac{d(S(x_0 | y_s) + ND^{(0)}(x_0))}{d\delta_0} &= \frac{\partial (U(x_0 | y_s) + \tau ND^{(0)}(x_0))}{\partial \delta_0} + \frac{\partial (S(x_0 | y_s) + ND^{(0)}(x_0))}{\partial y_s} \frac{\partial y_s}{\partial \delta_0} \\ &= \tau \frac{\partial ND^{(0)}(x_0)}{\partial \delta_0} + \frac{\partial ND^{(0)}(x_0)}{\partial y_s} \frac{\partial y_s}{\partial \delta_0} = \tau \frac{\partial ND^{(0)}(x_0)}{\partial \delta_0} + \frac{1}{(1 - \tau)} \frac{\partial U(x_0 | y_s)}{\partial y_s} \frac{\partial y_s}{\partial \delta_0}. \quad \mathbf{QED.} \end{aligned} \quad (68)$$

**Proof of Proposition 1:** Expression (36) is identical to (23), and expression (35) is obtained after substituting (33) into the expression for the debt value. Equations (31), (31), (32) and (33) are obtained developing (26), (28), (29) and (30), respectively, using notably (2).

Finally, equation (29),  $S^{(0)}(x_0 | y_s) = S_f^{(0)}(x_0 | y_f)$ , implies  $S^{(0)}(x | y_s) = S_f^{(0)}(x | y_f)$  for all  $x \geq y_f$ , given that  $S_f^{(0)}$  corresponds to the no-renegotiation case. But then,  $S^{(0)}(x | y_s) = S_f^{(0)}(x | y_f)$  for all  $x \geq y_f$  implies that no difference in the income flow stream between the two cases occurs before  $y_f$  is first reached, and consequently renegotiation only takes places after. **QED.**

**Proof of Proposition 2:** Consider the firm values when an *identical* coupon  $\delta_0$  is issued in (i) non-renegotiable (fully guaranteed) debt contracts and (ii) renegotiable (not fully guaranteed) debt contract. These are respectively, for  $x_t \geq y_f$ ,

$$S_f^{(0)}(x_t | y_f) + ND_f^{(0)}(x_t) = U(x_t | y_f) + \tau \frac{\delta_0}{\rho} (1 - \mathcal{P}(x_t \triangleright y_f)) + \tau V^*(y_f) \mathcal{P}(x_t \triangleright y_f), \quad (69)$$

$$S^{(0)}(x_t | y_s) + ND^{(0)}(x_t) = U(x_t | y_s) + \tau D^{(0)}(x_t) \quad (70)$$

$$\begin{aligned} &= U(x_t | y_f) + \tau \frac{\delta_0}{\rho} (1 - \mathcal{P}(x_t \triangleright y_f)) - (1 - \tau) V^*(y_f) \mathcal{P}(x_t \triangleright y_f) \\ &\quad + \left[ U(y_f | y_s) + \tau \frac{\delta_0}{\rho} (1 - \mathcal{P}(y_f \triangleright y_s)) \right] \mathcal{P}(x_t \triangleright y_f) \end{aligned} \quad (71)$$

$$\begin{aligned} &= U(x_t | y_f) + \tau \frac{\delta_0}{\rho} (1 - \mathcal{P}(x_t \triangleright y_f)) - (1 - \tau) V^*(y_f) \mathcal{P}(x_t \triangleright y_f) \\ &\quad + \left[ S^{(0)}(y_f | y_s) + ND^{(0)}(y_f) \right] \mathcal{P}(x_t \triangleright y_f). \end{aligned} \quad (72)$$

Now, from equation (29),  $S^{(0)}(x | y_s) = S_f^{(0)}(x | y_f)$  for all  $x \geq y_f$ , hence  $S^{(0)}(y_f | y_s) = 0$ . Using (70), the debt value is then equal to the unlevered firm value  $D^{(0)}(y_f) = U(y_f | y_s)/(1 - \tau)$ .

Therefore, the firm value at  $y_f$  is  $S^{(0)}(y_f | y_s) + ND^{(0)}(y_f) = U(y_f | y_s)/(1 - \tau) = \Pi(y_f) + [V^*(y_s) - \Pi(y_s)] \mathcal{P}(y_f \triangleright y_s)$ . Replacing in (72), the difference (72) - (69) is equal to

$$\left[ S^{(0)}(x_t | y_s) + ND^{(0)}(x_t) \right] - \left[ S_f^{(0)}(x_t | y_f) + ND_f^{(0)}(x_t) \right] = \quad (73)$$

$$\left( \Pi(y_f) - V^*(y_f) + [V^*(y_s) - \Pi(y_s)] \mathcal{P}(y_f \triangleright y_s) \right) \mathcal{P}(x_t \triangleright y_f), \quad (74)$$

which is strictly positive under Assumption 1. **QED.**

**Proof of Lemma 5:**

Suppose the firm issues debt  $(\delta_0, G_0^*(x))$  such that  $G_0^*(y_s) = 0$  as in (30) and the coupon  $\delta_0$  is chosen such that the firm is 100% levered at  $x_0$ , i.e.

$$\frac{\delta_0}{\rho}(1 - \mathcal{P}(x_0 \triangleright y_s)) = D^{(0)}(x_0) = U(x_0 | y_s) + \tau D^{(0)}(x_0)$$

Hence

$$U(x_0 | y_s) + \tau D^{(0)}(x_0) = \frac{U(x_0 | y_s)}{1 - \tau} = \Pi(x_0) + [V^*(y_s) - \Pi(y_s)] \mathcal{P}(x_0 \triangleright y_s) \quad (75)$$

By construction,  $S^{(0)}(x_0 | y_s) = 0$ . Now our finding that there cannot be debt renegotiation prior to  $y_f$ , together with equality (29), imply that

$$S_f^{(0)}(x_0 | y_f) = 0 \quad (76)$$

and hence  $y_f = x_0$ , since the equity value without renegotiation  $S_f^{(0)}(x_0 | y)$  is strictly concave in  $y$  (implied by Assumption 1) with a maximum at  $y_f$ .

Consider a single exchange offer strategy (as in the proof of Result 1) with renegotiation at  $\underline{x}_1 \leq x_0$ . There is zero collateral prior to renegotiation,  $G_0^*(y_s) = 0$ , and full collateral,  $G_1^*(y_s) = V^*(y_s)/N$  after the single exchange offer, hence  $\delta_1 < \delta_0$ , from Lemma 3. By construction of the single exchange offer strategy,  $y_s = y_f(\delta_1)$ . Hence  $\frac{dy_f(\delta_1)}{d\delta_1} > 0$  and  $y_f(\delta_0) = x_0$  imply that  $y_s < x_0$ . But then from (75),

$$D^{(0)}(x_0) = \Pi(x_0) + [V^*(y_s) - \Pi(y_s)] \mathcal{P}(x_0 \triangleright y_s) > V^*(x_0),$$

proving the claim, since  $V^*(x_0)$  is the debt capacity with a single creditor.

For the debt contract considered ( $G_0^*(y_s) = 0$  and leverage of 100%,  $y_s < x_0$ ) to be feasible, it must be true that  $\underline{x}_K < x_0$ , where  $\underline{x}_K$  being the point  $x$  of the last debt renegotiation round, i.e. renegotiation is delayed. But then note that in our single exchange offer strategy,  $\underline{x}_1 < x_0$ , otherwise either  $S^{(0)}(x_0 | y_s) > 0$  or  $S_f^{(0)}(x_0 | y_f) < 0$ , contradicting our starting assumption or (76). Thus,  $\underline{x}_K < x_0$ . **QED.**

**Proof of Proposition 3:** At the optimal capital, the firm with single-creditor debt is at its debt capacity  $\Lambda_s(x_0) = V^*(x_0)$ :

$$S_s(x_0 | \tilde{y}) + ND_s(x_0) = U(x_0 | \tilde{y}) + \tau V^*(x_0). \quad (77)$$

*First Part:* Following the proof of Proposition 2,

$$S^{(0)}(x_0 | y_s) + ND^{(0)}(x_0) > \left[ S_f^{(0)}(x_0 | y_f) + ND_f^{(0)}(x_0) \right] + \left( \Pi(y_f) - V^*(y_f) + [V^*(y_s) - \Pi(y_s)] \mathcal{P}(y_f \triangleright y_s) \right) \mathcal{P}(x_0 \triangleright y_f). \quad (78)$$

Now  $S_f^{(0)}(x_t | y_f) + ND_f^{(0)}(x_t) > U(x_0 | \tilde{y})$ , since  $S_f^{(0)}(x_t | y_f) + ND_f^{(0)}(x_t)$ , the value under the optimal capital structure with non-renegotiable debt, is larger than the value when non-renegotiable debt is kept to the level such that  $y_f = \tilde{y}$ , which in turn is larger than  $U(x_0 | \tilde{y})$ . Therefore

$$S^{(0)}(x_0 | y_s) + ND^{(0)}(x_0) > U(x_0 | \tilde{y}) + \left( \Pi(y_f) - V^*(y_f) + [V^*(y_s) - \Pi(y_s)] \mathcal{P}(y_f \triangleright y_s) \right) \mathcal{P}(x_0 \triangleright y_f). \quad (79)$$

Therefore, equations (77) and (79) yield:

$$V^*(x_0) \leq \frac{1}{\tau} \left( \Pi(y_f) - V^*(y_f) + [V^*(y_s) - \Pi(y_s)] \mathcal{P}(y_f \triangleright y_s) \right) \mathcal{P}(x_0 \triangleright y_f) \quad (80)$$

$$\Rightarrow S^{(0)}(x_0 | y_s) + ND^{(0)}(x_0) \geq S_s(x_0 | \tilde{y}) + ND_s(x_0). \quad (81)$$

*Second Part:* At the optimal capital with widely held renegotiable debt contracts

$$S^{(0)}(x_0 | y_s) + ND^{(0)}(x_0) = U(x_0 | y_s) + \tau N \frac{\delta_0}{\rho} [1 - \mathcal{P}(x_0 \triangleright y_s)]. \quad (82)$$

From (32),

$$\tau N D^{(0)}(x_0) = \tau \left( \frac{1 - \mathcal{P}(x_0 \triangleright y_s)}{1 - \mathcal{P}(y_f(\delta_0) \triangleright y_s)} \right) \left[ \Pi(y_f(\delta_0)) + (V^*(y_s) - \Pi(y_s)) \mathcal{P}(y_f(\delta_0) \triangleright y_s) \right]. \quad (83)$$

Therefore, given that  $U(x_0 | \tilde{y}) > U(x_0 | y_s)$ , equations (76), (82) and (83) yield:

$$V^*(x_0) > \left( \frac{1 - \mathcal{P}(x_0 \triangleright y_s)}{1 - \mathcal{P}(y_f(\delta_0) \triangleright y_s)} \right) \frac{U(y_f(\delta_0) | y_s)}{1 - \tau} \quad (84)$$

$$\Rightarrow S^{(0)}(x_0 | y_s) + ND^{(0)}(x_0) < S_s(x_0 | \tilde{y}) + ND_s(x_0). \quad \mathbf{QED.}$$

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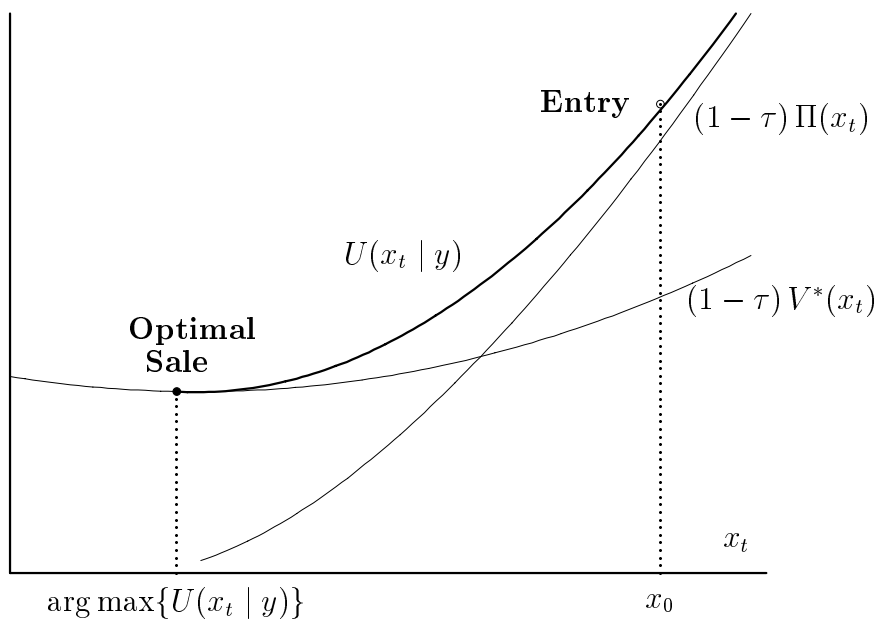


Figure 1: The Unlevered Firm



Table 1: Closed-Form Asset Pricing Formulas in the GBM-Linear Structure.

The table gives the expression of the constants  $(a_A, b_A, c_A)$  such that  $A(x_t) = a_A + b_A x_t + c_A x_t^\lambda$  and the relevant decision trigger levels.

$A(x_t)$		
$U(x_t   y)$	$a_A:$ $(1 - \tau) \Theta_0$ $b_A:$ $(1 - \tau) \Theta_1$ $c_A:$ $(1 - \tau) [\Theta_0^* - \Theta_0 + (\Theta_1^* - \Theta_1)y] y^{-\lambda}$	
	$\tilde{y}$	$[-\lambda/(1 - \lambda)] [(\Theta_0^* - \Theta_0) / (\Theta_1 - \Theta_1^*)]$
$S_f^{(0)}(x_t)$	$a_A:$ $(1 - \tau) [\Theta_0 - N\delta_0/\rho]$ $b_A:$ $(1 - \tau) \Theta_1$ $c_A:$ $(1 - \tau) [-\Theta_0 - \Theta_1 y_f + N\delta_0/\rho] y_f^{-\lambda}$	
$D_f^{(0)}(x_t)$	$a_A:$ $\delta_0/\rho$ $b_A:$ $0$ $c_A:$ $[(\Theta_0^* + \Theta_1^* y_f)/N - \delta_0/\rho] y_f^{-\lambda}$	
	$y_f$	$[-\lambda/(1 - \lambda)] [(N\delta_0/\rho - \Theta_0) / \Theta_1]$
$S^{(0)}(x_t)$	$a_A:$ $(1 - \tau) [\Theta_0 - N\delta_0/\rho]$ $b_A:$ $(1 - \tau) \Theta_1$ $c_A:$ $(1 - \tau) [\Theta_0^* - \Theta_0 + N\delta_0/\rho + (\Theta_1^* - \Theta_1)y_s] y_s^{-\lambda}$	
$D^{(0)}(x_t)$	$a_A:$ $\delta_0/\rho$ $b_A:$ $0$ $c_A:$ $[-\delta_0/\rho] y_s^{-\lambda}$	
$S_s^{(0)}(x_t)$	$a_A:$ $(1 - \tau) [\Theta_0 - N\delta_0/\rho]$ $b_A:$ $(1 - \tau) \Theta_1$ $c_A:$ $(1 - \tau) [-\Theta_0^* - \Theta_1^* x_s + N\delta_0/\rho + \{\Theta_0^* - \Theta_0 + (\Theta_1^* - \Theta_1)\tilde{y}\} (x_s/\tilde{y})^\lambda] x_s^{-\lambda}$	
$D_s^{(0)}(x_t)$	$a_A:$ $\delta_0/\rho$ $b_A:$ $0$ $c_A:$ $[(\Theta_0^* + \Theta_1^* x_s)/N - \delta_0/\rho] x_s^{-\lambda}$	
	$x_s$	$[-\lambda/(1 - \lambda)] [(N\delta_0/\rho - \Theta_0^*) / \Theta_1^*]$

Table 2: Type of Debt and Optimal Contract.

For each types of debt, the table exhibits the (a) optimal debt contract and (b) optimal capital structure at entry, and the resulting (c) firm value, (d) credit spreads, (e) risk premium, (f) first renegotiation trigger levels and (g) liquidation trigger levels. Input parameters are  $\mu = 0.03$ ,  $\sigma = 0.30$ ,  $\theta_1 = 1$ ,  $\theta_1^* = 0.1$ ,  $\theta_0 = -4$ ,  $\theta_0^* = 4$ ,  $\rho = 0.06$ ,  $N = 10$  and  $\tau = 0.13$ .

(a) Optimal Debt Contract Issued at $x_0 = 30$ :		Value
Dispersed Creditors + Optimal Guaranteeing	[ Coupon $\delta_0$	0.1803
	[ Slope $dG_0^*(y_s)/dy_s$	0.2017
Non-Renegotiable Debt (Fully Guaranteed)	Coupon $\delta_{0,f}$	0.0680
Single Creditor	Coupon $\delta_{0,s}$	0.0600
(b) Optimal Leverage at $x_0 = 30$ :		Value
Dispersed Creditors + Optimal Guaranteeing	$l^{(0)}(x_0) \equiv ND^{(0)}(x_0)/V^{(0)}(x_0)$	80.42 %
Non-Renegotiable Debt (Fully Guaranteed)	$l_f^{(0)}(x_0) \equiv ND_f^{(0)}(x_0)/V_f^{(0)}(x_0)$	40.04 %
Single Creditor	$l_s(x_0) \equiv ND_s(x_0)/V_s(x_0)$	29.11 %
Relative difference	$l_f^{(0)}(x_0)/l^{(0)}(x_0) - 1$	-50.20 %
Relative difference	$l_s(x_0)/l^{(0)}(x_0) - 1$	-63.80 %
(c) Firm Value at $x_0 = 30$ :		Value
Dispersed Creditors + Optimal Guaranteeing	$V^{(0)}(x_0) \equiv S^{(0)}(x_0) + ND^{(0)}(x_0)$	24.95
Non-Renegotiable Debt (Fully Guaranteed)	$V_f^{(0)}(x_0) \equiv S_f^{(0)}(x_0) + ND_f^{(0)}(x_0)$	24.12
Single Creditor	$V_s(x_0) \equiv S_s(x_0) + ND_s(x_0)$	24.05
Relative difference	$V_f^{(0)}(x_0)/V^{(0)}(x_0) - 1$	-3.32 %
Relative difference	$V_s(x_0)/V^{(0)}(x_0) - 1$	-3.62 %
(d) Credit Spreads at $x_0 = 30$ :		Value (bps)
Dispersed Creditors + Optimal Guaranteeing	$s(x_0) \equiv \delta_0/D^{(0)}(x_0) - \rho$	298.54
Non-Renegotiable Debt (Fully Guaranteed)	$s_f(x_0) \equiv \delta_{0,f}/D_f^{(0)}(x_0) - \rho$	104.38
Single Creditor	$s_s(x_0) \equiv \delta_{0,s}/D_s(x_0) - \rho$	257.14
Relative difference	$s_f(x_0)/s^{(0)}(x_0) - 1$	-65.04 %
Relative difference	$s_s(x_0)/s^{(0)}(x_0) - 1$	-13.87 %
(e) Risk Premium at $x_0 = 30$ :		Value/ $\delta$
Dispersed Creditors + Optimal Guaranteeing	$p(x_0) \equiv \delta_0 - \rho D^{(0)}(x_0)$	33.22 %
Non-Renegotiable Debt (Fully Guaranteed)	$p_f(x_0) \equiv \delta_{0,f} - \rho D_f^{(0)}(x_0)$	14.82 %
Single Creditor	$p_s(x_0) \equiv \delta_{0,s} - \rho D_s(x_0)$	30.00 %
Relative difference	$p_f(x_0)/p^{(0)}(x_0) - 1$	-55.39 %
Relative difference	$p_s(x_0)/p^{(0)}(x_0) - 1$	-9.71 %
(f) First Renegotiation trigger levels:		Value
Dispersed Creditors + Optimal Guaranteeing	$y_f(\delta_0)$	17.02
Non-Renegotiable Debt (Fully Guaranteed)		n.a.
Single Creditor	$x_s$	30.00
(g) Liquidation trigger levels:		Value
Dispersed Creditors + Optimal Guaranteeing	$y_s$	9.97
Non-Renegotiable Debt (Fully Guaranteed)	$y_f$	7.66
Single Creditor (same as Unlevered Firm)	$\tilde{y}$	4.44